



# APECE-302: Radio & Television Engineering

## Applied Physics, Electronics & Communication Engineering

Lecture # 06



University of  
Dhaka | APECE  
DU

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# Random Variable

## ❑ Function

- ❑ Domain: a sample space
- ❑ Range: some set of real numbers

$$\boxed{\text{CDF:}} \quad F_X(x) = P(X \leq x)$$

$$\boxed{\text{PDF:}} \quad f_X(x) = \frac{d}{dx} F_X(x)$$

## ❑ Properties of pdf

- ❑ Integration over the whole  $x$ ?
- ❑ At least equal 0

# Joint cdf and pdf

❑ Joint cdf

$$F(x, y) = P(X \leq x, Y \leq y).$$

❑ Joint pdf

❑ Marginal densities

$$f_{X,Y}(x, y) = f_{Y|X}(y|x)f_X(x) = f_{X|Y}(x|y)f_Y(y)$$

❑ Conditional pdf

❑ Statistically independent

$$f_{X,Y}(x, y) = f_X(x) \cdot f_Y(y)$$

# Statistical Averages

- Expected value or mean

$$E[X] = \int_{-\infty}^{\infty} x f(x) dx.$$

- Function of a RV

$$E(g(X)) = \int_{-\infty}^{\infty} g(x) f(x) dx.$$

- Moments and Central Moments

$$\mu_k = E[(X - E[X])^k] = \int_{-\infty}^{+\infty} (x - \mu)^k f(x) dx.$$

- Chebyshev Inequality

$$\Pr(|X| \geq \varepsilon) \leq \frac{E X^2}{\varepsilon^2}$$

# Statistical Averages

## □ Characteristic Function:

$$\phi_X(\vartheta) = E[\exp(j\vartheta X)] = \int_{-\infty}^{+\infty} f_X(x) \exp(j\vartheta x) dx$$

$$f_X(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_X(\vartheta) \exp(-j\vartheta x) d\vartheta$$

## □ Characteristic F. of GRV

$$\phi_X(\vartheta) = \exp\left(j\vartheta \mu_X - \frac{1}{2} \vartheta^2 \sigma_X^2\right)$$

# Statistical Averages

- ❑ Joint Moments

$$E[X^i Y^j] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^i y^j f_{X,Y}(x,y) dx dy$$

- ❑ Correlation  $i = j = 1$

- ❑ Covariance Correlation of Centered RV

- ❑ Correlation coefficients

$$\rho = \frac{\text{cov}[XY]}{\sigma_X \sigma_Y}$$

- ❑ Uncorrelated

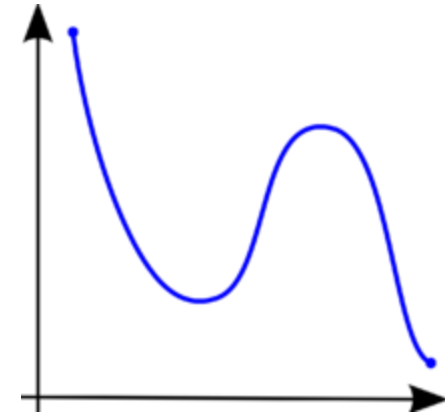
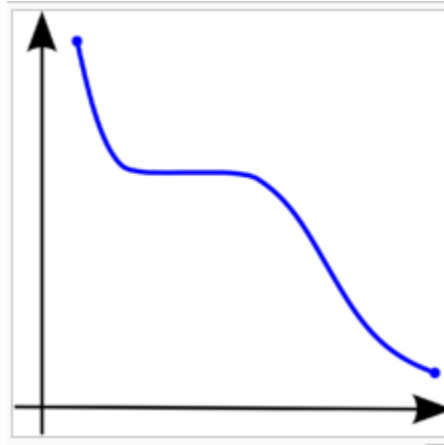
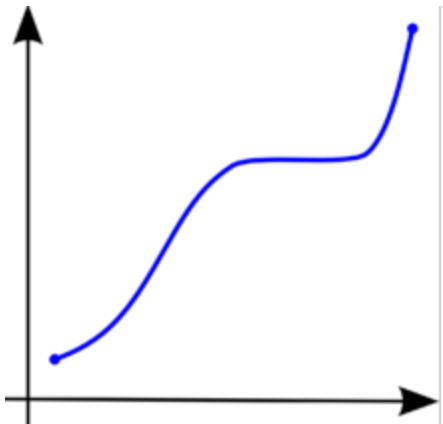
$$E\{xy\} = E\{x\}E\{y\}$$

- ❑ Orthogonal

$$E\{xy\} = 0$$

# Transformation of RV

- $Y=g(x)$
- Monotone transformations



- Conservation of prob.

$$f_Y(y)|dy| = f_X(x)|dx|$$



# Transformations of RV

$$f_Y(y) = \frac{f_X(x)}{\left| \frac{dy}{dx} \right|} = \frac{f(x)}{\left| \frac{dg}{dx} \right|} \Bigg|_{x=g^{-1}(y)}$$

□ Many-to-one Transformation

$$f_Y(y) = \sum_k \frac{f(x_k)}{\left| \frac{dg}{dx_k} \right|} \Bigg|_{x_k=g^{-1}(y)}$$

# Q & A

