Chapter 5
Digital Modulation Techniques

Collected;
Available @:
http://web.ee.ccu.edu.tw/~wl/

- Main difference between the analog modulation and digital modulation:

- Digital modulation system represents a small set of abstract symbols, e.g., 0 and 1 for a binary transmission system.

- Analog modulation system represents a continuous waveform.

- To transmit a digital message, a digital modulation allocates a piece of time called signal interval and generates a continuous function that represents the symbol.

- The message signal is often transformed onto a baseband signal.

- In a wireless communication system, a second part of the modulator converts the baseband signal to a radio-frequency (RF) signal, modulating the phase, frequency, or amplitude of the carrier.

- A digital demodulator:

  it outputs a decided symbol.

- An analog demodulator:

  it produces an output that approximately equals to the message signal.

5.1 Baseband Pulse Transmission

- Baseband pulse transmission:

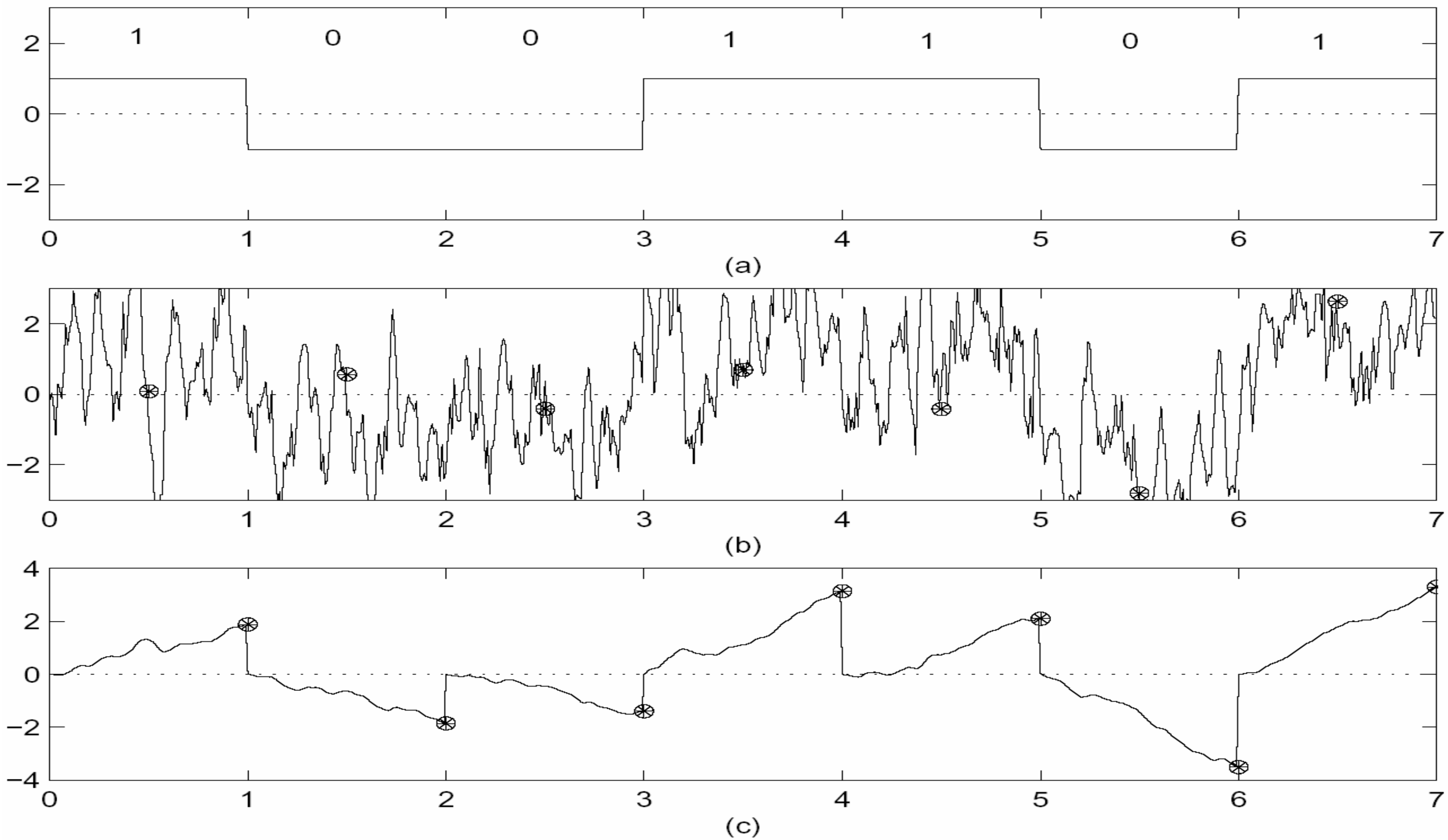
  a transmission technique which does not require carrier modulation.

- The digital information for the baseband pulse transmission is transformed to a pulse train.

- Symbols 1 and 0 are represented by positive and negative rectangular pulses of equal amplitude and equal duration T_b.

- Fig. 5.1-1(a):

  the pulse is a rectangular pulse, and the binary information stream of “**1001101**” is transformed onto a baseband signal in where T_b is assumed to be 1 sec.



- Fig. 5.1-1 Signals for baseband pulse transmission. (a) baseband pulse signal, (b) the received signal corrupted by noise and the sampled points, (c) the output of the correlator and the corresponding sampling points.

- Let *m* be the transmitted symbol, i.e., $m \in \{0,1\}$
  transmitted signal denoted as:

$$s(t) = \begin{cases} +A & \text{if} \quad m = 1 \\ -A & \text{if} \quad m = 0 \end{cases} \qquad (5.1\text{-}1)$$

  for $0 \le t \le T_b$.

- In this case, $1/T_b$ is called the bit rate.

- Given the received signal, the receiver is required to make a decision in each signaling interval as to whether the transmitted symbol is a 1 or a 0.

- Recover the original digital stream:

  1. sample the received signal at the sampling rate $1/T_b$.

  2. based on the sampled value, the decision device is used to "guess" the transmitted symbol.

- Short distance wired transmission,
  e.g., RS232 standard:

  If the sampled value is positive, then it decides that a 1 was transmitted.

  If the sampled value is negative, then it decides that a 0 was transmitted.

  → it is not suitable for long distance transmission systems.

- Long distance transmissions:

  Noise will add to the transmitted signal.

- Fig. 5.1-1(b):

  It gives a noise version of the received signal and the corresponding sampled points that are denoted by circles.

- Sampled value is directly used to decide which symbol was transmitted.

- The decisions made based on the sampled values produce the output "**1101001**" which contains two errors as compared with the transmitted sequence "**1001101**".

- It is very possible that the sampled value goes to the opposite polarity at the sampling instance.

- Fig. 5.1-1 (c) :
- Assume the starting and ending times of the pulse are known.
- Compare the area of the received signal-plus-noise waveform by integrating the received signal over the T_b-second signaling interval.
- The decisions based on these sampled values are "**1001101**" which contains no error as compared with the transmitted sequence.

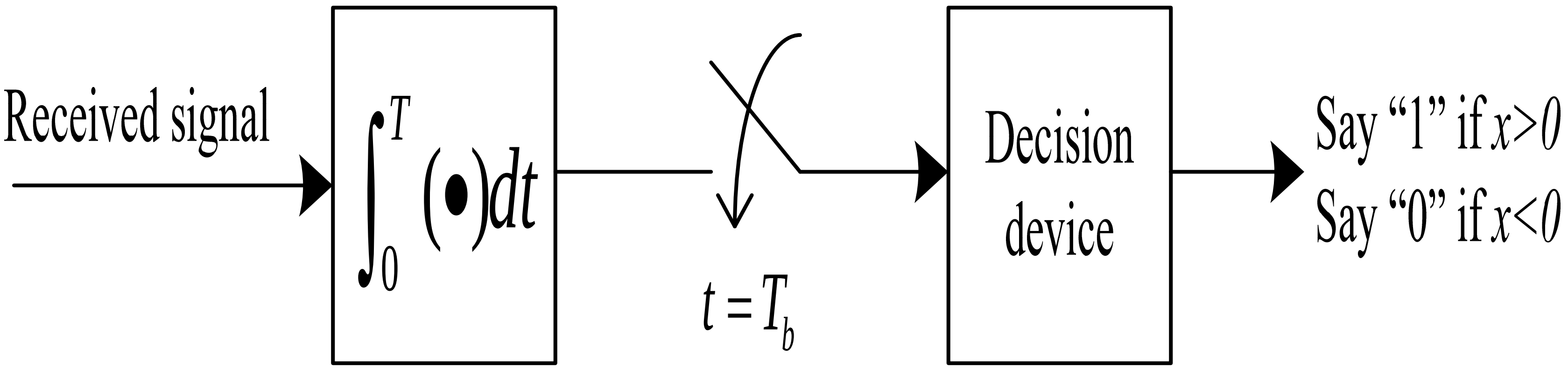


Fig. 5.1-2 A receiver for baseband pulse transmission.

- The integrator will integrate the waveform over $(0, T_b)$ with the output being sampled at time T_b .
- the integrate-and-dump device is called a "***correlator***".

- A noise component is present at the output of the integrator.
- Since the additive noise is always assumed to be of zero mean, it takes on positive and negative values with equal probability.
- Noise will be “averaged” over by the integrator, and hence the noise component will be suppressed by the integrator.

- The integrator can be thought as an energy collector that collects the energy of the received signal.
- Based on the sampled values, the decision device makes a decision on the transmitted symbol.
- If the output of the correlator at the sampling instance is positive, it decides that a 1 was transmitted.

- It is very important to determine the required bandwidth for the transmitted signal.
- For broadband transmission, if the signal has a bandwidth which is greater than the bandwidth of the channel, the transmitted signal will not be able to transmit through the channel without distortion.

- The digital signaling is transmitted consecutively one-by-one.

- We express the transmitted signal as:

$$s(t) = \sum_{k=-\infty}^{\infty} b_k h(t - kT_b) \qquad (5.1\text{-}2)$$

where $b_k \in \{+1, -1\}$ for binary transmission and $h(t)$ is a pulse shaping function.

$$h(t) = \begin{cases} 1 & 0 \le t < T \\ 0 & \text{otherwise} \end{cases} \qquad (5.1\text{-}3)$$

Power Spectral Density (PSD)

- In communication systems, we have to know how the transmitted power is distributed over the frequencies.

- The question is what the power distribution of the signal $s(t)$ is.

- A formal explanation of the PSD of $s(t)$ requires the knowledge of random process.

- The meaning of the PSD is similar to the Fourier transform, we may explain it intuitively.

- Let $s_T(t)$ be a truncated signal of $s(t)$ given by:

$$s_T(t) = \begin{cases} s(t) & -T/2 \le t \le T/2 \\ 0 & \text{otherwise} \end{cases} \qquad (5.1\text{-}4)$$

- The PSD for $s(t)$ is defined as

$$P_s(f) = \lim_{T \to \infty} \frac{1}{T} |S_T(f)|^2 \qquad (5.1\text{-}5)$$

where $S_T(f)$ is the Fourier transform of $s_T(t)$.

- The power of the signal $s(t)$ is defined as

$$P_s = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |s(t)|^2 \, dt \qquad (5.1\text{-}6)$$

- **Parseval's theorem**:

$$\int_{-T/2}^{T/2} |s(t)|^2 \, dt = \int_{-\infty}^{\infty} |S_T(f)|^2 \, df \qquad (5.1\text{-}7)$$

$$
\begin{aligned}
P_s &= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |s(t)|^2 dt \\
&= \lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{\infty} |S_T(f)|^2 df \qquad (5.1\text{-}8) \\
&= \int_{-\infty}^{\infty} \left(\lim_{T \to \infty} \frac{|S_T(t)|^2}{T} \right) df
\end{aligned}
$$

- Substituting (5.1-5) into (5.1-8):

$$
P_s = \int_{-\infty}^{\infty} P_s(f) df \qquad (5.1\text{-}9)
$$

- The power of $s(t)$ is exactly equal to the integration of the PSD over frequency. This is why $P_s(f)$ is called PSD of $s(t)$.

- From Equation (5.1-2), the signal $s(t)$ is a combination of the same waveform $h(t)$ of different delays and amplitudes.

- Let $H(f)$ be the Fourier transform of $h(t)$. The PSD $P_s(f)$ of $s(t)$ can be shown:

$$P_s(f) = \frac{1}{T_b} |H(f)|^2 \qquad (5.1\text{-}10)$$

- In this case, $h(t)$ is a rectangular pulse, according to Equation (3.3-19), the Fourier transform of $h(t)$ is

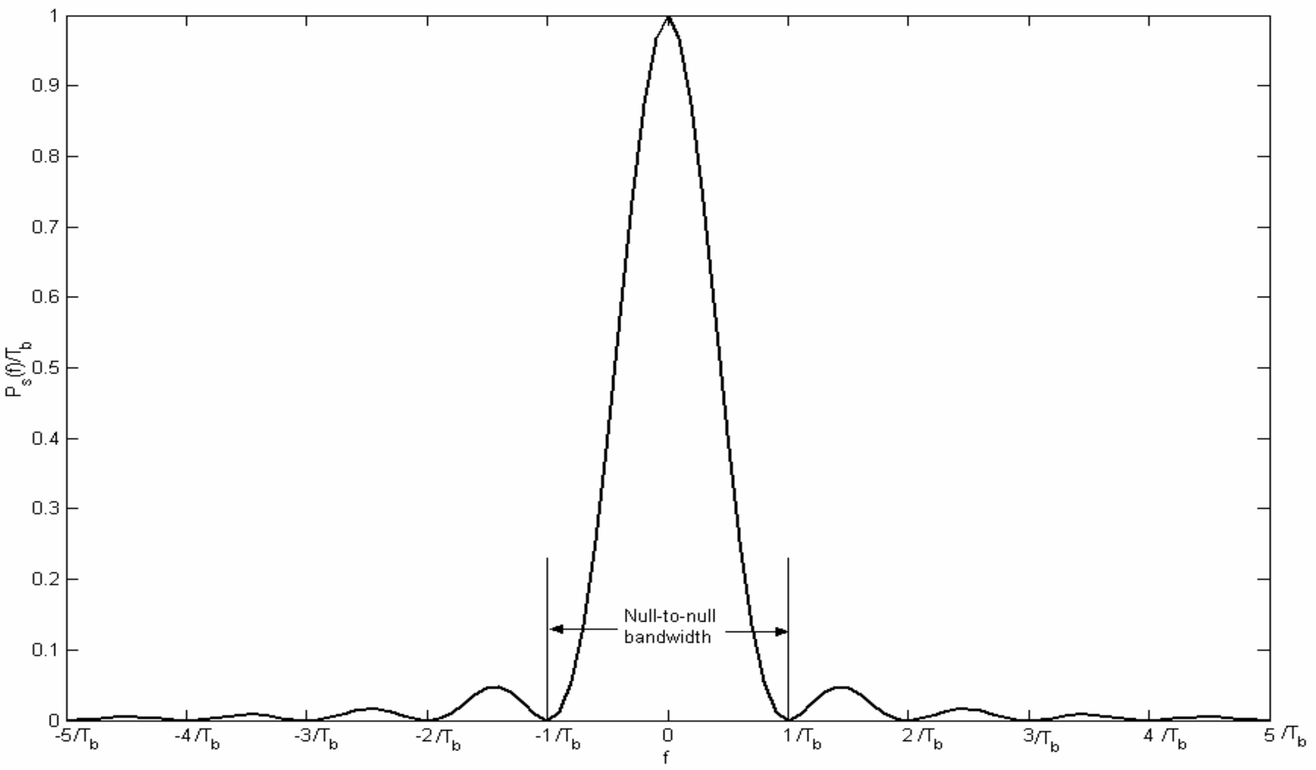
$$H(f) = e^{-j\pi f T_b} \frac{\sin(\pi f T_b)}{\pi f} = e^{-j\pi f T_b} T_b \operatorname{sinc}(fT_b) \qquad (5.1\text{-}11)$$

where $\mathrm{sinc}(x) = \dfrac{\sin(\pi x)}{\pi x}$ (5.1-12)

- Therefore, we have $|H(f)| = T_b \mathrm{sinc}(fT_b)$ and the PSD $P_s(f)$ of $s(t)$ is:

$$P_s(f) = T_b \mathrm{sinc}^2(fT_b)$$ (5.1-13)

- Figure 5.1-3 plots the PSD of $s(t)$ with $T_b = 1$.



- Figure 5.1-3 The PSD of the transmitted baseband signal $s(t)$.

- Figure 5.1-3:
- Most of the power of $s(t)$ concentrates over $[-1/T_b, 1/T_b]$.

- PSD is zero at $-1/T_b$ and $1/T_b$.
  →define $2/T_b$ is the bandwidth of the baseband signal.

- $2/T_b$ also called the null-to-null bandwidth.

- Identify the differences between ***data rate*** and ***signal bandwidth***:

- **Data rate**:

  defined as the number of bits transmitted per second.

- For example:

  a data rate of 1 Mbits/sec means that the system is capable of transmitting 1 Mega bits every second.

- **Signal bandwidth**:

  defined as the bandwidth for which most of the signal power is concentrate on.

  define 1:

  the width of the spectrum for which 99% of the power of the signal is concentrated on.

  or define 2:

  simply use the null-to-null bandwidth.

- Bandwidth is proportional to the bit rate.
- If the bit rate is high, T_b is small and the bandwidth becomes very large.
- The communication system handling high data rate transmission must be able to cope with a wide range of frequencies.

- Where more than one users are using the channel, the baseband pulse transmission method does not work because there is no way to distinguish these users.

5.2 Amplitude-Shift Keying (ASK)

- We will add carrier waves into the modulation scheme.
- ASK is the simplest modulation technique.
- ASK is modulated over the carrier's amplitude, similar to the amplitude modulation for analog modulation.

- Binary transmission: we require two waveforms $s_1(t)$ and $s_2(t)$.

- If the transmitted symbol is "1",
  $s_1(t)$ is used over the signaling interval $(0, T_b)$.
  If the transmitted symbol is "0",
  $s_2(t)$ is used over the signaling interval $(0, T_b)$.

- Assume the probabilities of transmitting a "1" and a "0" are equal.

- For ASK, the transmitted waveforms can be expressed as

$$s_1(t) = \sqrt{\frac{4E_b}{T_b}} \cos(2\pi f_c t) \qquad (5.2\text{-}1)$$
$$s_2(t) = 0$$

for $0 \le t \le T_b$, where E_b is the averaged transmitted signal energy per bit and f_c is the carrier frequency which is equal to n_c / T_b for some fixed integer n_c.

- The averaged transmitted energy which is expressed as follows:

$$0.5\int_0^{T_b} s_1^2(t)dt + 0.5\int_0^{T_b} s_2^2(t)dt \qquad (5.2\text{-}2)$$

- (5.2-2) is equal to E_b .

- The transmitted signal $s(t)$ can be expressed as

$$s(t) = \begin{cases} s_1(t) & \text{for symbol "1"} \\ s_2(t) & \text{for symbol "0"} \end{cases} \qquad (5.2\text{-}3)$$

for $0 \le t \le T_b$.

- ASK is often referred to as *on-off keying*.

- Fig. 5.2-1(a) gives a transmitted waveform resulting from the digital transmission of "**1001101**", where $E_b = 1$, $f_c = 5$ Hz, and $T_b = 1$ sec.

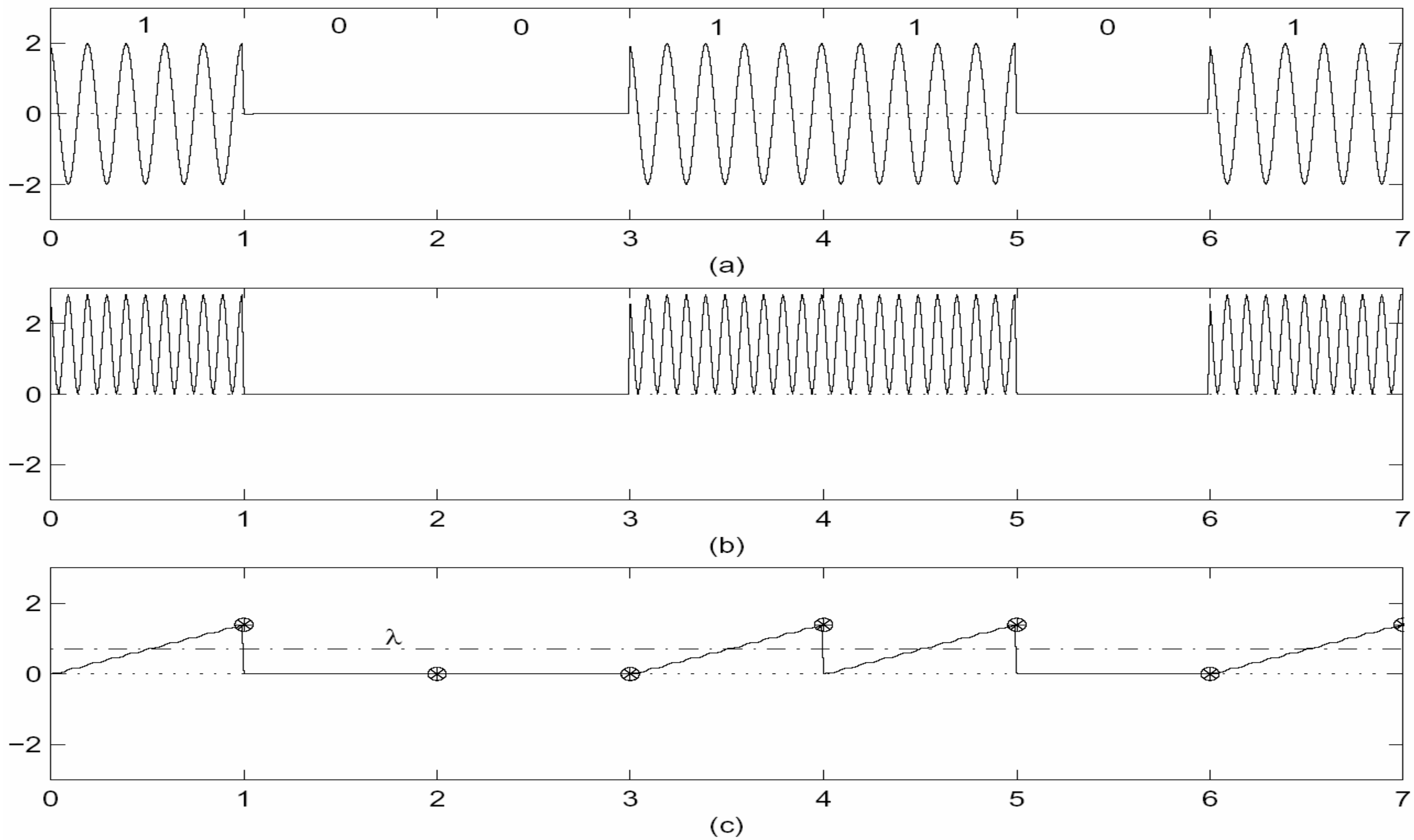
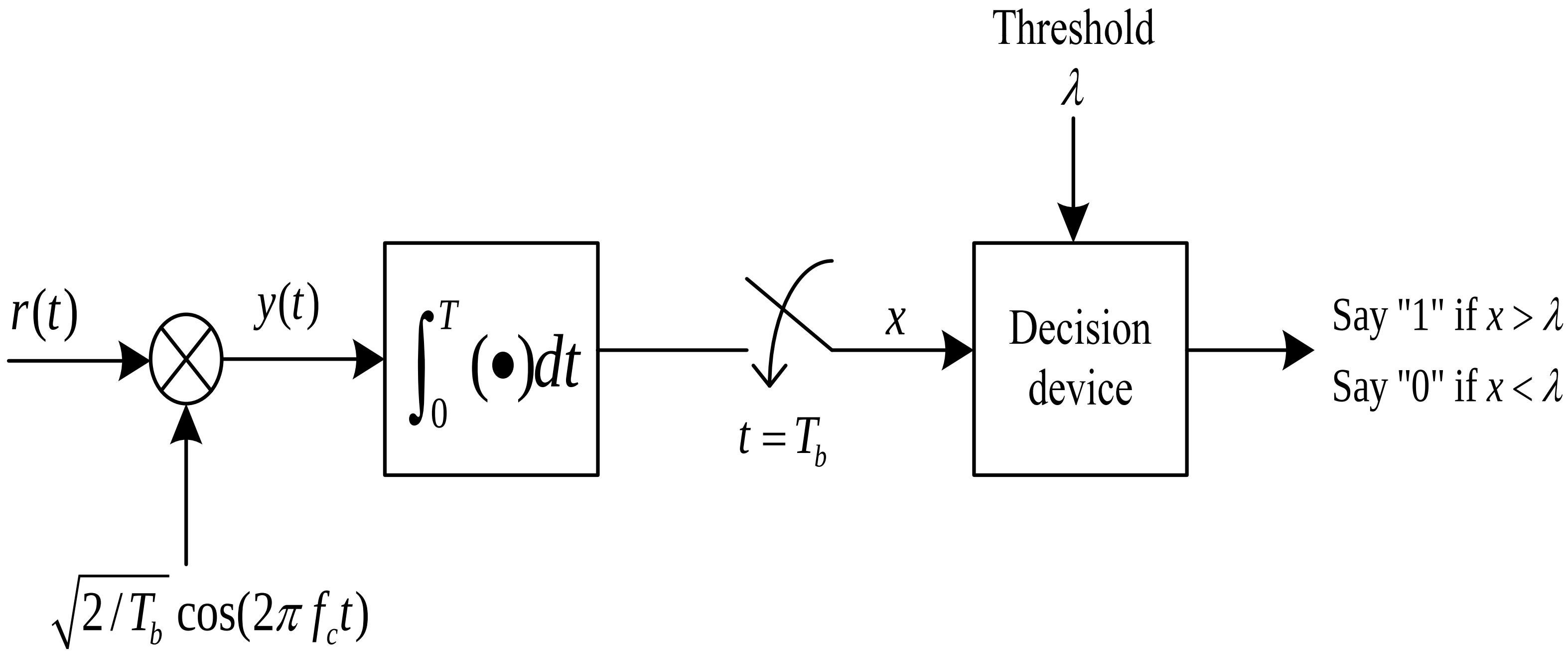


Fig. 5.2-1 Signals for ASK, (a) transmitted signal, (b) the signal $y(t) = s(t) \times \sqrt{2/T_b} \cos(2\pi f_c t)$, (c) the output of the integrator and the corresponding sampling points.

- Use Fig. 5.2-2. to explain how the demodulation works.



- Fig. 5.2-2 A receiver for amplitude-shift keying

- Assuming that the received signal is noise free and multiplied by a unit-energy signal $\sqrt{2/T_b}\cos(2\pi f_c t)$.

- The expression after multiplication:

$$y(t) = s(t) \times \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) \qquad (5.2\text{-}4)$$

- Fig. 5.2-1(b) shows the corresponding signal $y(t)$.

- If

$$s(t) = \sqrt{\frac{4E_b}{T_b}} \cos(2\pi f_c t) \quad (5.2\text{-}5),$$

we have

$$y(t) = \frac{\sqrt{8E_b}}{T_b} \cos^2(2\pi f_c t) \qquad (5.2\text{-}6)$$

- $y(t) \ge 0$ for the corresponding period.

- An integrator is then employed to integrate the signal $y(t)$ over $[0, T_b]$ with the output being sampled at time T_b .

- Defining the sampled signal as x :

$$x = \int_0^{T_b} \sqrt{\frac{4E_b}{T_b}} \cos(2\pi f_c t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) dt$$

$$= \frac{\sqrt{8E_b}}{T_b} \int_0^{T_b} \frac{1 + \cos(4\pi f_c t)}{2} dt. \qquad (5.2\text{-}7)$$

- Replacing $f_c = n_c / T_b$ in (5.2-7):

$$
\begin{aligned}
x &= \frac{\sqrt{8E_b}}{T_b}\left(\int_0^{T_b}\frac{1}{2}dt + \int_0^{T_b}\frac{1}{2}\cos\left(4\pi\frac{n_c}{T_b}t\right)dt\right)\\
&= \frac{\sqrt{8E_b}}{T_b}\left(\frac{1}{2}t\Big|_0^{T_b} + \frac{1}{2}\frac{1}{4\pi(n_c/T_b)}\sin\left(4\pi\frac{n_c}{T_b}t\right)\Big|_0^{T_b}\right)\\
&= \sqrt{2E_b}.
\end{aligned}
\qquad (5.2\text{-}8)
$$

The second term in the brackets is zero.

- If $s_2(t)$ was transmitted, the sampled signal is *x* = 0 since $s_2(t) = 0$.

- In summary:

$$x = \begin{cases} \sqrt{2E_b} & \text{if 1 is transmitted} \\ 0 & \text{if 0 is transmitted} \end{cases} \qquad (5.2\text{-}9)$$

- Fig. 5.2-1(c):
- gives the resulting signal from the integrator and the corresponding sampled points.

- The decision device based on the sampled values makes a decision of “**1001101**” which is exactly the same as the transmitted sequence.

- Although there are only two possible outcomes for the noiseless case, the actual received signal may be corrupted by noise.

- For certain systems, we have infinitely many outcomes for the output of the correlator.

  → decision device require a **threshold value** λ to distinguish which symbol was transmitted.

- Threshold value is set to the center of 0 and $\sqrt{2E_b}$, i.e., $\lambda = \sqrt{E_b/2}$, so that minimum bit error probability can be achieved.

- If , $x > \lambda$ the decision device makes a decision of 1.
- If , $x < \lambda$ the decision device makes a decision of 0.

- The detection method requires a carrier with the same frequency and phase as those of the carrier in the transmitter.

  →Those kinds of receivers are called ***coherent detector***.

- Without the knowledge of the carrier frequency and phase

  →***noncoherent*** *detector*.

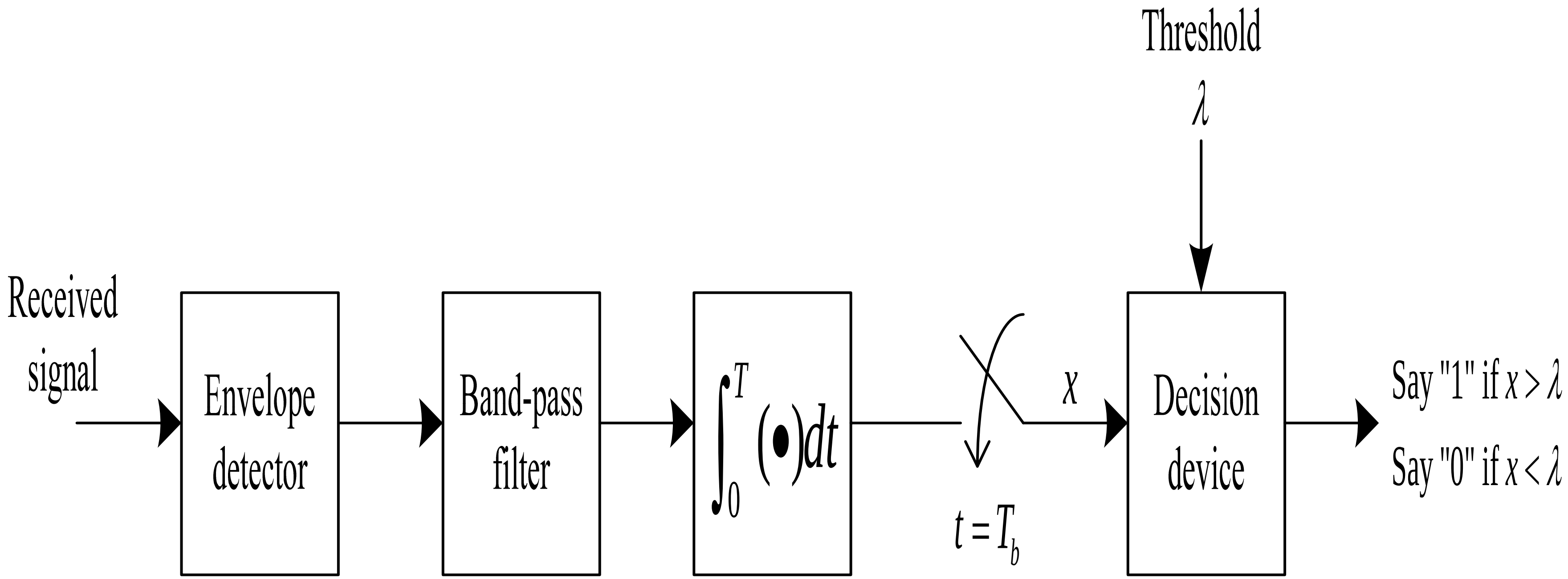


Fig. 5.2-3 **Noncoherent detector** for amplitude-shift keying

- Fig. 5.2-3:
- The envelope detector is used to recover the envelope of the carrier wave.
- The band-pass filter is used to remove the out-of-band noise.
- The receiver is not optimal since an envelope detector will enhance the noise at the input of the correlator and hence increase the bit error probability.

- In general, the transmitted energy for the noncoherent detector must be doubled to achieve the same bit error probability as that of the coherent detector.
- The collection of all possible signal points is called the signal constellation.
- For binary ASK:

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos\left(2\pi f_c t\right) \qquad (5.2\text{-}10)$$

which is a unit energy signal over (0, T_b).

- All the possible signal $s_i(t)$ for i = 1, 2, can be represented as

$$s_i(t) = s_{i1}\phi_1(t) \qquad (5.2\text{-}11)$$

$s_{11} = \sqrt{2E_b}$ and $s_{21} = 0$.

- The geometric interpretation of these two signal points can be illustrated in Fig. 5.2-4.

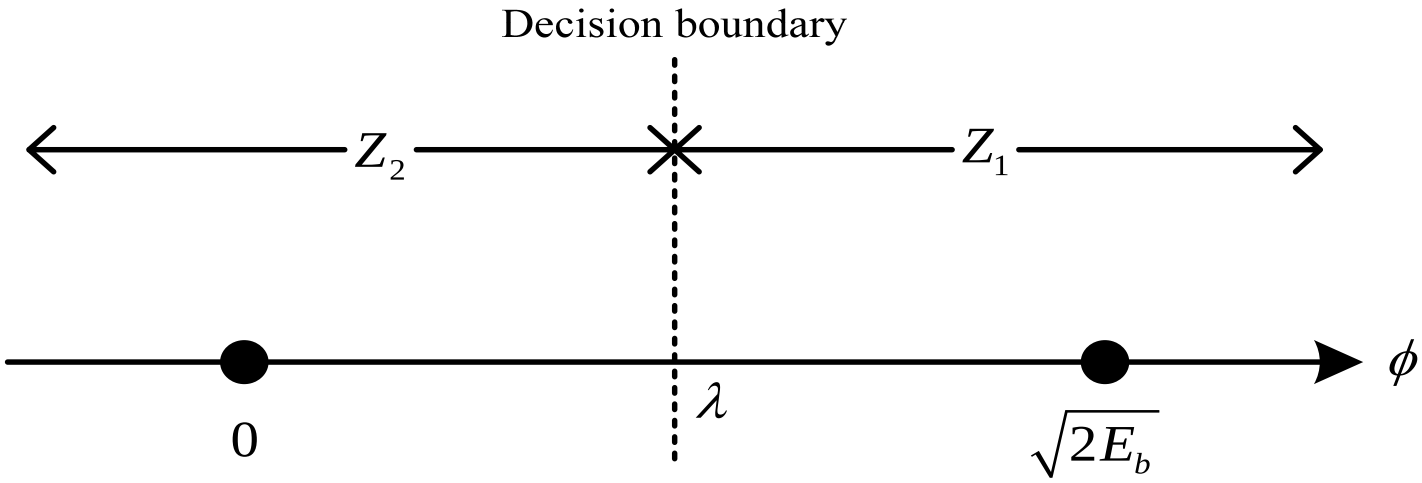


Fig. 5.2-4

Geometric representation of binary ASK signals.

- The decision boundary is determined by the threshold value λ .

- If x lies in the region Z_1, then a decision of a "1" is made.

  If x lies in the region Z_2, then a decision of a "0" is made.

- One advantage in using the signal space representation is that it is much easier to identify the "**distance**" between signal points.

- The distance between signal points is closely related to the symbol error rate of a given constellation.

- Ex: ASK

  the distance between two signal points is

  $\sqrt{2E_b}$

- Increase the transmitted energy of the signals, i.e. E_b, can reduce the probability of detection error.

- The distance between two signal points will be increased which makes the received signal point less probable be located in the wrong region.

- Increasing E_b is equivalent to increasing the magnitude of the signal $s_1(t)$.

multi-dimensional

- For the *N*-dimensional case, a set of **orthonormal** functions $\{\phi_1(t), \phi_2(t), \ldots, \phi_N(t)\}$ is required to present all the possible transmitted signals $\{s_1(t), s_2(t), \ldots, s_M(t)\}$.
- M = 2 for binary case.

- Transmitted signal can be presented as a linear combination:

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t) \qquad (5.2\text{-}12)$$

for *i*= 1,…, *M* and $0 \le t \le T$

- And

$$\int_0^T \phi_i(t)\,\phi_j(t)\,dt = \begin{cases} 1 & i = j \\ 0 & i \ne j \end{cases} \qquad (5.2\text{-}13)$$

- We may plot the signal vector $s_i = [s_{i1}, s_{i2}, \cdots; s_{i\mathbf{N}}]$ as a point in the *N*-dimensional signal space to represent the time-domain signal $s_i(t)$.

5.3 Binary Phase-Shift Keying (BPSK)

- In a BPSK system, we use a pair of signals $s_1(t)$ and $s_2(t)$ to represent binary symbols 1 and 0, respectively, as follows:

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \qquad (5.3\text{-}1)$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \qquad (5.3-2)$$

where $0 < t < T_b$, E_b is the transmitted energy per bit, and f_c is the carrier frequency which is chosen to be equal to n_c / T_b for some fixed integer n_c.

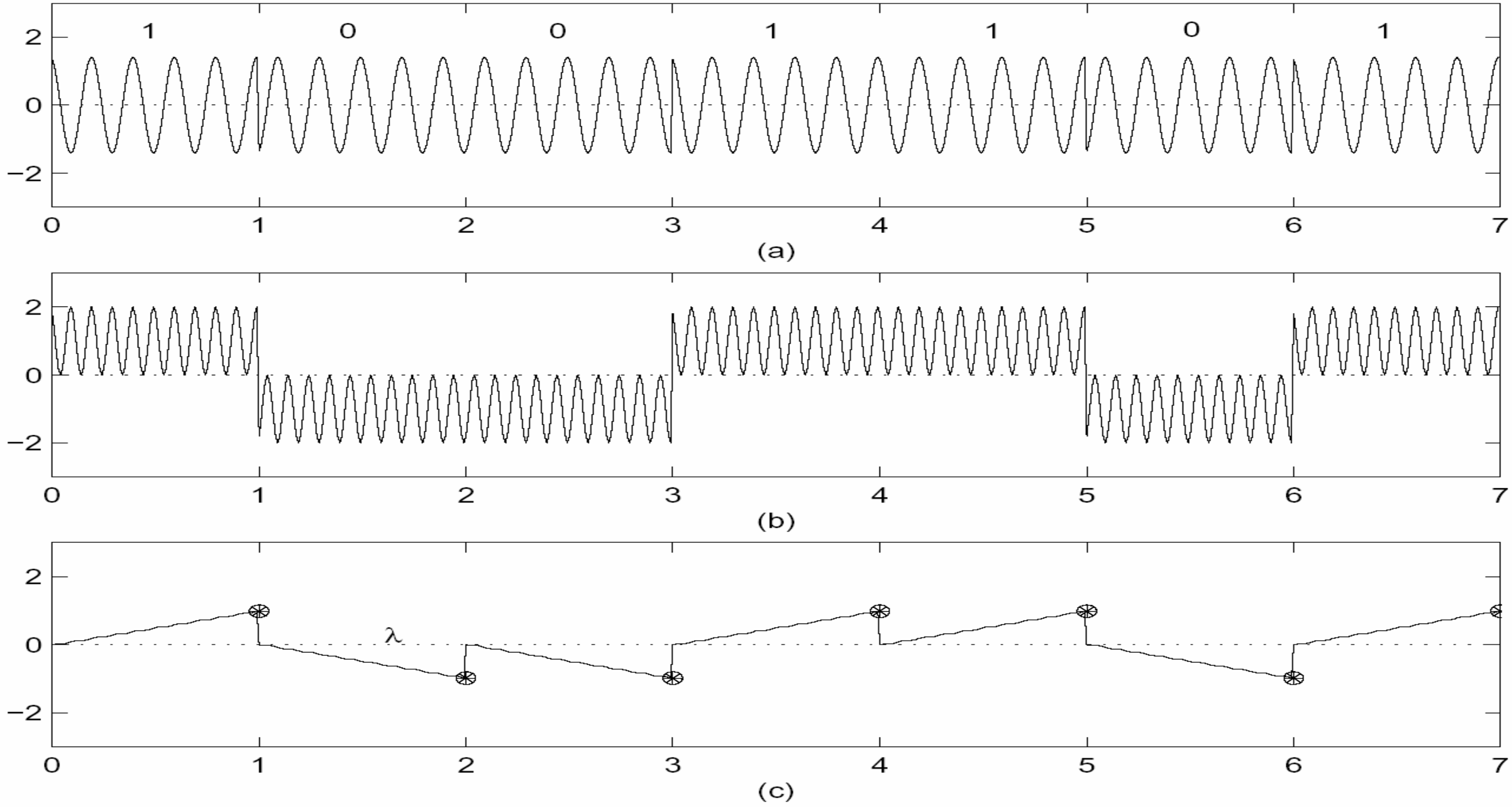
- The averaged transmitted signal energy which is expressed as

$$0.5\int_0^{T_b} s_1^2(t)\,dt + 0.5\int_0^{T_b} s_2^2(t)\,dt = E_b$$

- A pair of sinusoidal waves that differ on in a relative phase shift of 180 degrees are referred to as antipodal signals.

- Fig. 5.3-1(a):

  A transmitted signal when the binary stream is "**1001101**" where $E_b = 1$, $f_c = 5$ Hz, and $T_b = 1$ sec.



- Fig. 5.3-1 Signals of BPSK, (a) the transmitted signal, (b) the signal $r(t)\sqrt{1/2}\cos(2\pi f_c t)$, (c) the output of the correlator and the corresponding sampling points.

- Fig. 5.3-2:

  An optimal detector for BPSK system.

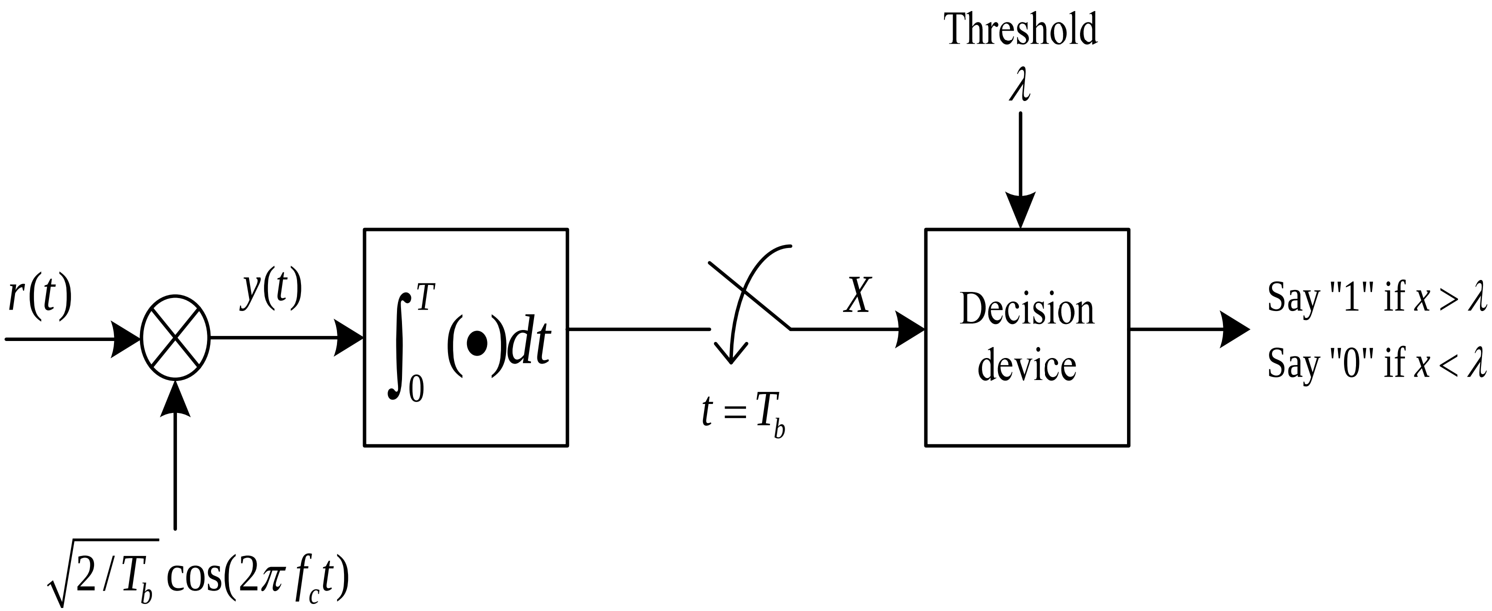


Fig. 5.3-2 A coherent detector for BPSK.

- Fig. 5.3-2:

  The received signal is first multiplied by the unit energy ***truncated*** signal $\phi_1(t) = \sqrt{2/T_b}\cos(2\pi f_c t)$ if $0 \le t \le T_b$ and $\phi_1(t) = 0$ if otherwise.

- Assuming the received signal is noise-free, we have the following expression after multiplication

$$y(t) = s(t) \times \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) \qquad (5.3\text{-}3)$$

- Therefore,

$$y(t)=\begin{cases}\dfrac{2\sqrt{E_b}}{T_b}\cos^2(2\pi f_c t) & \text{if 1 was transmitted}\\ -\dfrac{2\sqrt{E_b}}{T_b}\cos^2(2\pi f_c t) & \text{if 0 was transmitted}\end{cases} \qquad (5.3\text{-}4)$$

- Fig. 5.3-1 (b):
  An example of the signal *y*(*t*).

- The two possible outcomes from the output of the correlator for noiseless case can be shown to be:

$$x = \begin{cases} +\sqrt{E_b} & \text{if 1 was transmitted} \\ -\sqrt{E_b} & \text{if 0 was transmitted} \end{cases} \qquad (5.3\text{-}5)$$

- The threshold value λ in this case must be set to 0 which corresponds to the center between $-\sqrt{E_b}$ and $\sqrt{E_b}$.

- Fig. 5.3-1 (c):

  Plot the output signal of the integrator and the corresponding sampling points.

- If the sampled value is greater then zero, a decision of 1 is made.

  If the sampled value is less than zero,

  a decision of 0 is made.

- The decision device based on the sampled values makes a decision of "**1001101**" which is exactly the same as the transmitted sequence.

- For geometric representation, $s_1(t)$ and $s_2(t)$ can be expressed in terms of $\phi_1(t) = \sqrt{2/T_b}\cos(2\pi f_c t)$ as follows:

$$s_1(t) = \sqrt{E_b}\,\phi_1(t), \qquad 0 \le t \le T_0$$
$$s_2(t) = -\sqrt{E_b}\,\phi_1(t), \qquad 0 \le t \le T_0. \qquad (5.3\text{-}6)$$

- Fig. 5.3-3:

  A coherent BPSK system is characterized by having a signal space that is one-dimensional, with a signal constellation consisting of two message points.

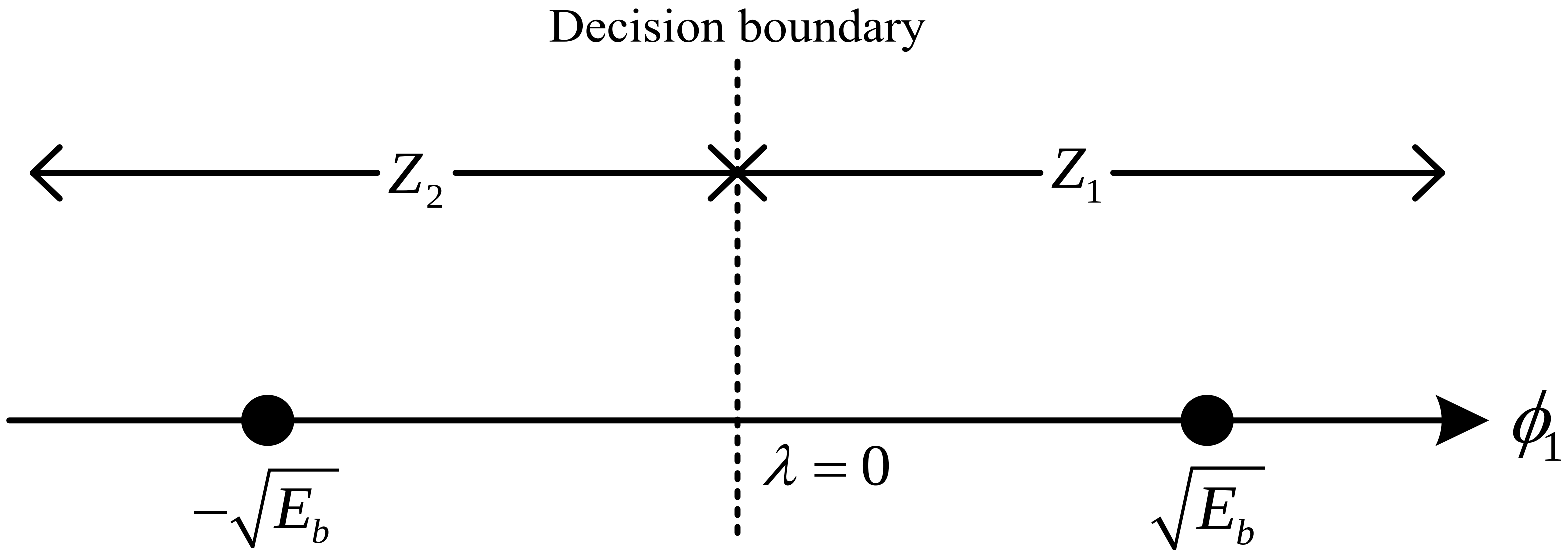


Fig. 5.3-3 Signal-space diagram for coherent BPSK.

- The decision boundary is determined by the threshold value λ which in this case is equal to 0.

- If the received signal x lies in region Z_1, then a decision of 1 is made.

  If the received signal x lies in region Z_2, then a decision of 0 is made.

- The transmitted BPSK signal can be expressed as:

$$s(t) = \sqrt{E_b} \sum_{k=-\infty}^{\infty} b_k \phi_1(t - kT_b) \qquad (5.3\text{-}7)$$

where $b_k \in \{+1, -1\}$

- $\phi_1(t)$ is actually a truncated cosine waveform as shown in Figure 5.3-4.

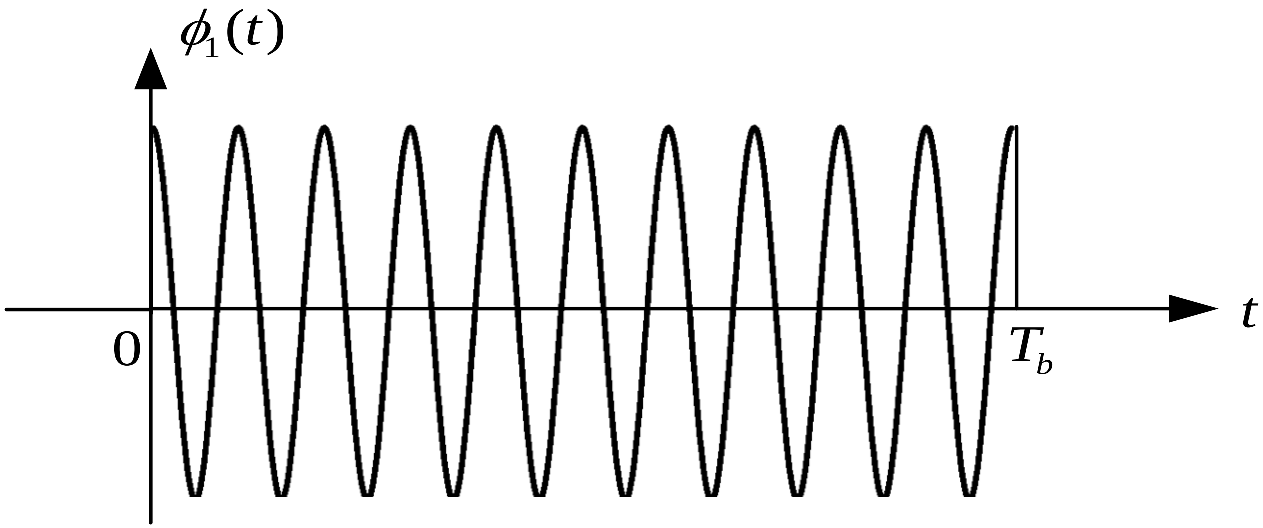


Fig. 5.3-4 A truncated cosine waveform $\phi_1(t)$.

- Let $h(t) = \sqrt{E_b}\phi_1(t)$.
- The PSD for $s(t)$ can be calculated from Equation (5.1-10)

$$P_s(f) = \frac{1}{T_b} |H(f)|^2 \qquad (5.3\text{-}8)$$

- $\phi_1(t)$ can be expressed as:

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \Pi\left(\frac{t - T_b/2}{T_b}\right)\cos(2\pi f_c t) \qquad (5.3\text{-}9)$$

where $\Pi(x)$ is called the rectangular function and defined as follows:

$$\Pi(x) = \begin{cases} 1 & \text{if } -\frac{1}{2} \le x \le \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

so $\Pi\left(\frac{t - T_b/2}{T_b}\right) = 1$ for $0 \le t \le T_b$,

and $\Pi\left(\frac{t - T_b/2}{T_b}\right) = 0$, otherwise.

- Fourier transform of the **product** of two functions in time-domain is equivalent to the **convolution** of the Fourier transforms of these two functions.

$$H(f) = \sqrt{E_b} F\left[\Pi\left(\frac{t - T_b/2}{T_b} \right) \right] * F[\cos(2\pi f_c t)] \quad (5.3\text{-}11)$$

- From Equation (3.3-19), the Fourier transform of the rectangular function is:

$$F\left[\Pi\left(\frac{t-T_b/2}{T_b}\right)\right]=T_b\mathrm{sinc}(T_b f)e^{-j\pi T_b f} \quad \text{(5.3-12)}$$

- From Equation (3.5-11), we have

$$F[\cos(2\pi f_c t)]=\frac{1}{2}\delta(f-f_c)+\frac{1}{2}\delta(f+f_c) \quad \text{(5.3-13)}$$

- We can see:

$$H(f)=\sqrt{\frac{E_b}{4}}T_b\text{sinc}(T_b(f-f_c))e^{-j\pi T_b(f-f_c)}+\sqrt{\frac{E_b}{4}}T_b\text{sinc}(T_b(f+f_c))e^{-j\pi T_b(f+f_c)}$$

(5.3-14)

- Assume $f_c >> 1/T_b$. Then

$$|H(f)|^2=\sqrt{\frac{E_b}{4}}T_b^2\text{sinc}^2(T_b(f-f_c))+\sqrt{\frac{E_b}{4}}T_b^2\text{sinc}^2(T_b(f+f_c))$$

(5.3-15)

- The PSD is:

$$P_s(f) = \frac{1}{T_b} |H(f)|^2$$

$$= \sqrt{\frac{E_b}{4}} T_b \operatorname{sinc}^2(T_b(f - f_c)) + \sqrt{\frac{E_b}{4}} T_b \operatorname{sinc}^2(T_b(f + f_c)) \qquad (5.3\text{-}16)$$

- The PSD for the case with $E_b = 1$, $T_b = 1\mu s$ (data rate is $R = 1/T_b = 1$ Mbit/sec) and $f_c = 6\text{MHz}$ is illustrated in Fig. 5.3-5.

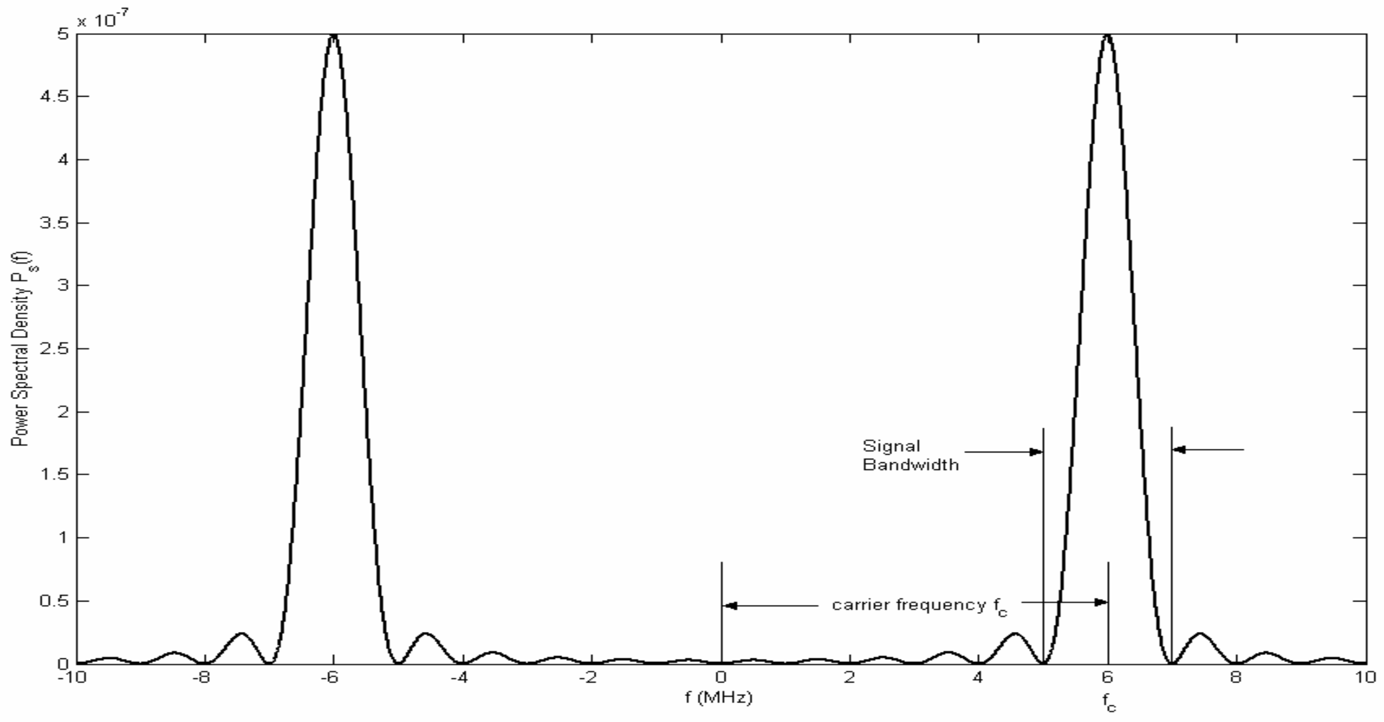


Fig. 5.3-5 The PSD for a BPSK signal.

- The main role of the signal concentrates over the carrier frequency f_c =6MHz.
- The bandwidth of the signal is 2MHz.

- We use a carrier frequency to transmit the signal.
- The signal bandwidth is determined by the data rate.
- The higher the data rate, the smaller T_b and the larger $2/T_b$.

5.4 Binary Frequency-Shift Keying (FSK)

- In a binary FSK system, symbols 1 and 0 are distinguished from each other by transmitting one of two sinusoidal waves that differ in frequency by a fixed amount.

- A typical pair of sinusoidal waves is:

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t)$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t) \qquad (5.4\text{-}1)$$

for $0 \le t < T_b$, where E_b is the transmitted signal energy per bit.

- The transmitted frequency is:

  $f_i = \dfrac{n_c + i}{T_b}$ for some fixed integer n_c

  and i = 1, 2. (5.4-2)

- Note: $f_2 - f_1 = 1/T_b$

- $s_1(t)$ ← symbol 1

  $s_2(t)$ ← symbol 0

- $s_1(t)$ and $s_2(t)$ are orthogonal.

- Define: $\phi_i(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_i t)$

  for i = 1, 2 and $0 \leq t < T_b$ (5.4-3)

- $\phi_1(t)$ and $\phi_2(t)$ are **orthonormal** basis functions.

- Show that $\phi_1(t)$ and $\phi_2(t)$ have unit energy:
- The energy of the signal $\phi_i(t)$ over $(0, T_b)$ is given by:

$$\int_0^{T_b} \phi_i^2(t)dt = \frac{2}{T_b}\int_0^{T_b} \cos^2(2\pi f_i t)dt$$

$$= \frac{2}{T_b}\left(\int_0^{T_b} \frac{1}{2}dt + \int_0^{T_b} \frac{1}{2}\cos(4\pi f_i t)dt\right)$$

$$= \frac{2}{T_b}\left(\frac{t}{2}\bigg|_0^{T_b} + \frac{1}{8\pi f_i}\sin(4\pi f_i t)\bigg|_0^{T_b}\right). \qquad (5.4\text{-}4)$$

- Substituting $f_i = (n_c + i)/T_b$ into the above equation, we have:

$$\int_0^{T_b} \phi_i^2(t)dt = \frac{2}{T_b}\left(\left(\frac{T_b}{2} - 0\right) + \frac{1}{8\pi(n_c+i)/T_b}\left(\sin\left(4\pi\frac{n_c+i}{T_b}T_b\right) - 0\right)\right)$$

$$= \frac{2}{T_b}\left(\frac{T_b}{2} + \frac{1}{8\pi(n_c+i)/T_b}\sin(4(n_c+i)\pi)\right)$$

$$= 1. \qquad (5.4\text{-}5)$$

- The last equality follows since $n_c + i$ is a positive integer.

→the second term in the brackets is zero.

$$\int_0^{T_b} \phi_1(t)\phi_2(t)dt = \frac{2}{T_b}\int_0^{T_b} \cos(2\pi f_1 t)\cos(2\pi f_2 t)dt$$

$$= \frac{1}{T_b}\int_0^{T_b} \cos(2\pi(f_1 + f_2)t) + \cos(2\pi(f_1 - f_2)t)dt.$$

(5.4-6)

- Substituting $f_i = (n_c + i)/T_b$ into the above equation:

$$\int_0^{T_b} \phi_1(t)\phi_2(t)dt = \frac{1}{T_b}\left(\int_0^{T_b} \cos\left(2\pi\frac{(2n_c + 3t)}{T_b}\right)dt + \int_0^{T_b} \cos\left(2\pi\frac{1}{T_b}t\right)dt\right).$$

(5.4-7)

- Since $2n_c$+1 is a positive integer, the first integration integrates the cosine term over exactly $2n_c + 1$ periods.

  → the first term is zero.

- The second term is integrated over one period of a cosine signal and hence the result is also zero.

- $$\therefore \int_0^{T_b} \phi_1(t)\phi_2(t)\,dt = 0. \qquad (5.4\text{-}8)$$

- Let $f_0 = 1/T_b$.
- By Equation (5.4-2), we have
  $f_2 = (n_c + 2) f_0$ and $f_1 = (n_c + 1) f_0$
- By using Equations (3.1-6) and (3.1-7), we can conclude that $\phi_1(t)$ and $\phi_2(t)$ are **orthonormal**.

- To generate a binary FSK signal:

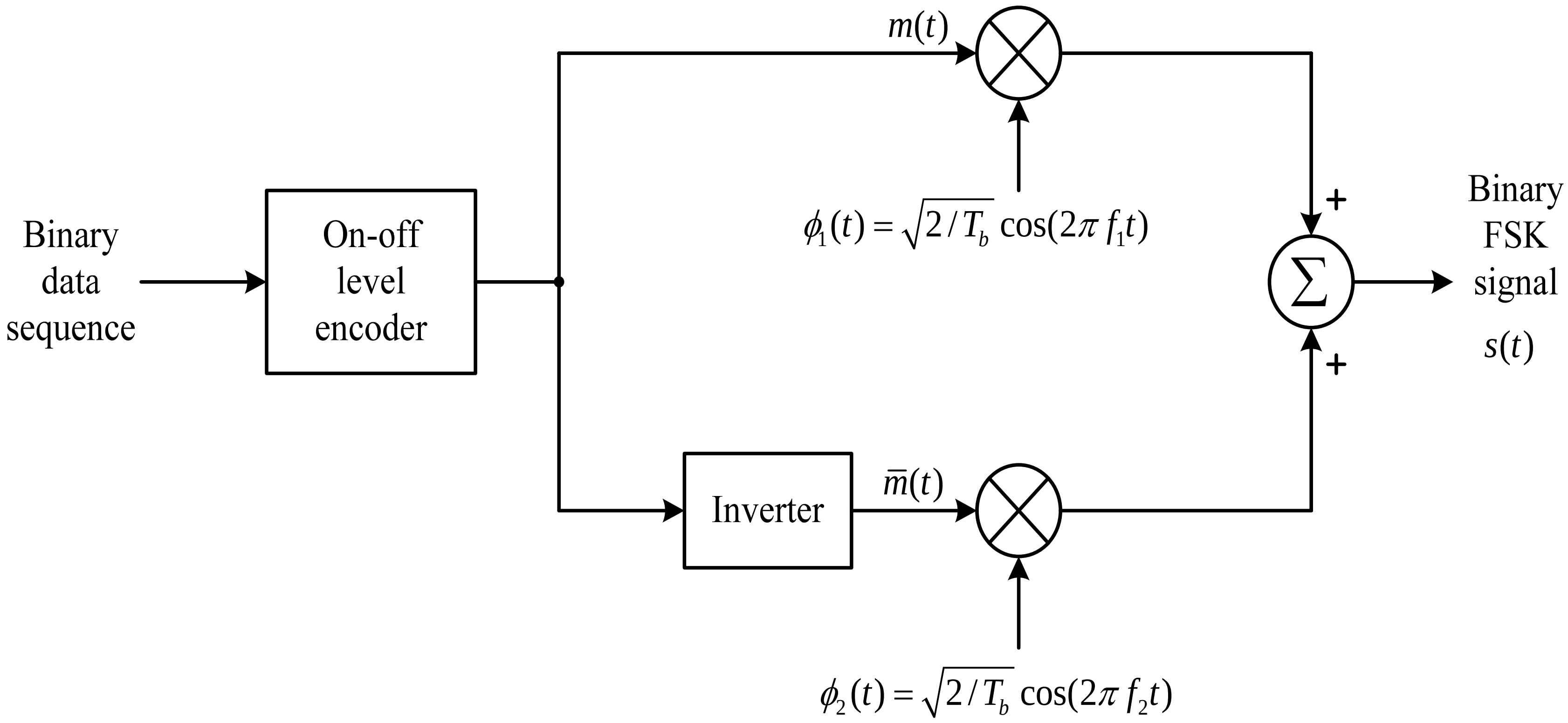


Fig. 5.4-1 A modulator for an FSK system.

- Fig. 5.4-1:
- symbol 1 is represented by a constant amplitude of $\sqrt{E_b}$.
  symbol 0 is represented by zero.

- If the transmitted symbol is 1, the carrier of the upper channel will be turned on, and that of the lower channel will be turned off.

- If the transmitted symbol is 0, the carrier of the upper channel will be turned off, and that of the lower channel will be turned on.

- The resulting output signal is altered between two carriers of different frequencies controlled by the input symbol.

- Fig. 5.4-2 (a) gives an example of resulting waveform when the input binary stream is "**1001101**", where $E_b = 1$, $T_b = 1$sec, $f_1 = 3$ Hz, and $f_2 = 4$ Hz.

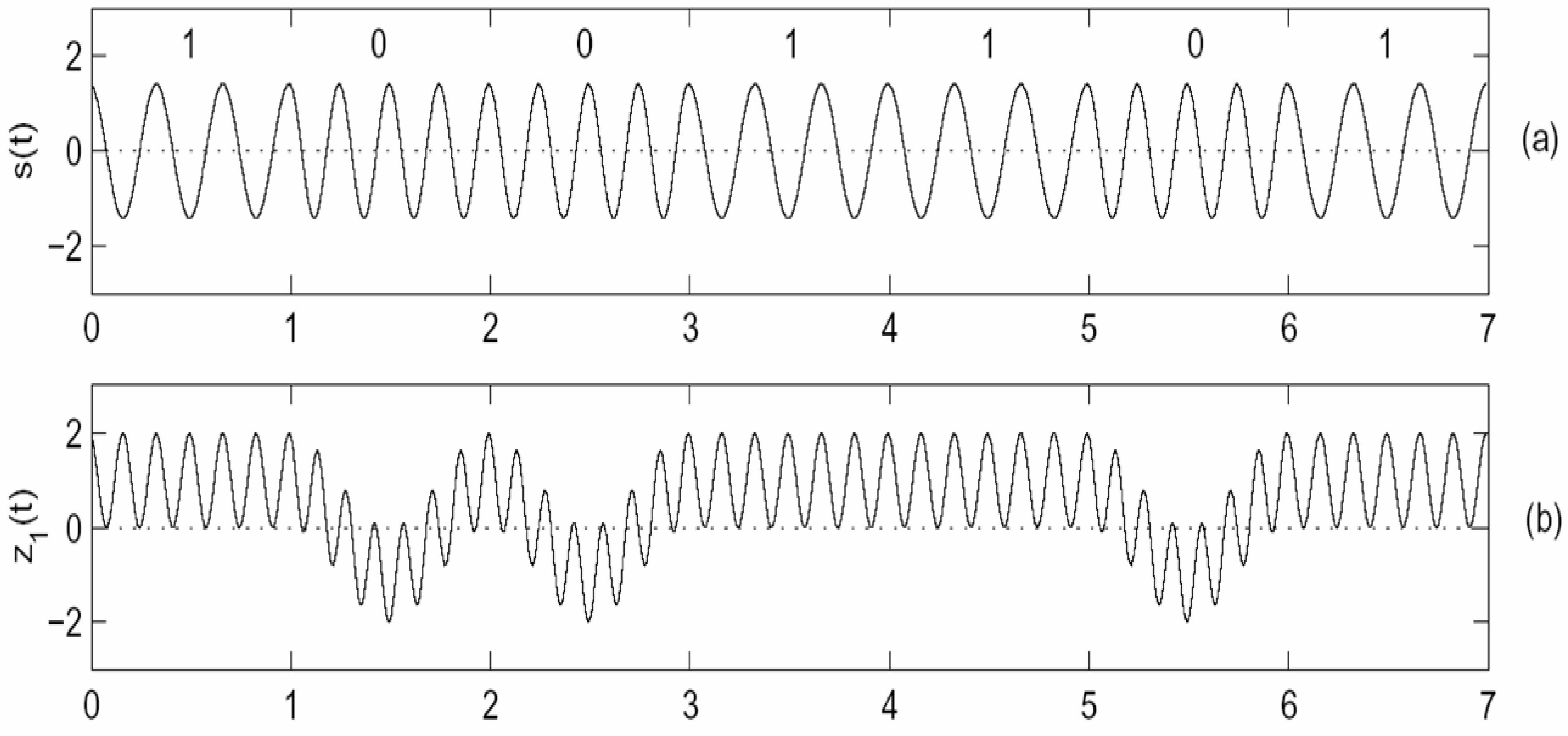


Fig. 5.4-2 Signals for an FSK system, (a) the transmitted signal, (b) the multiplication output

$$z_1(t) = s(t)\sqrt{2/T_b}\cos(2\pi f_1 t)$$

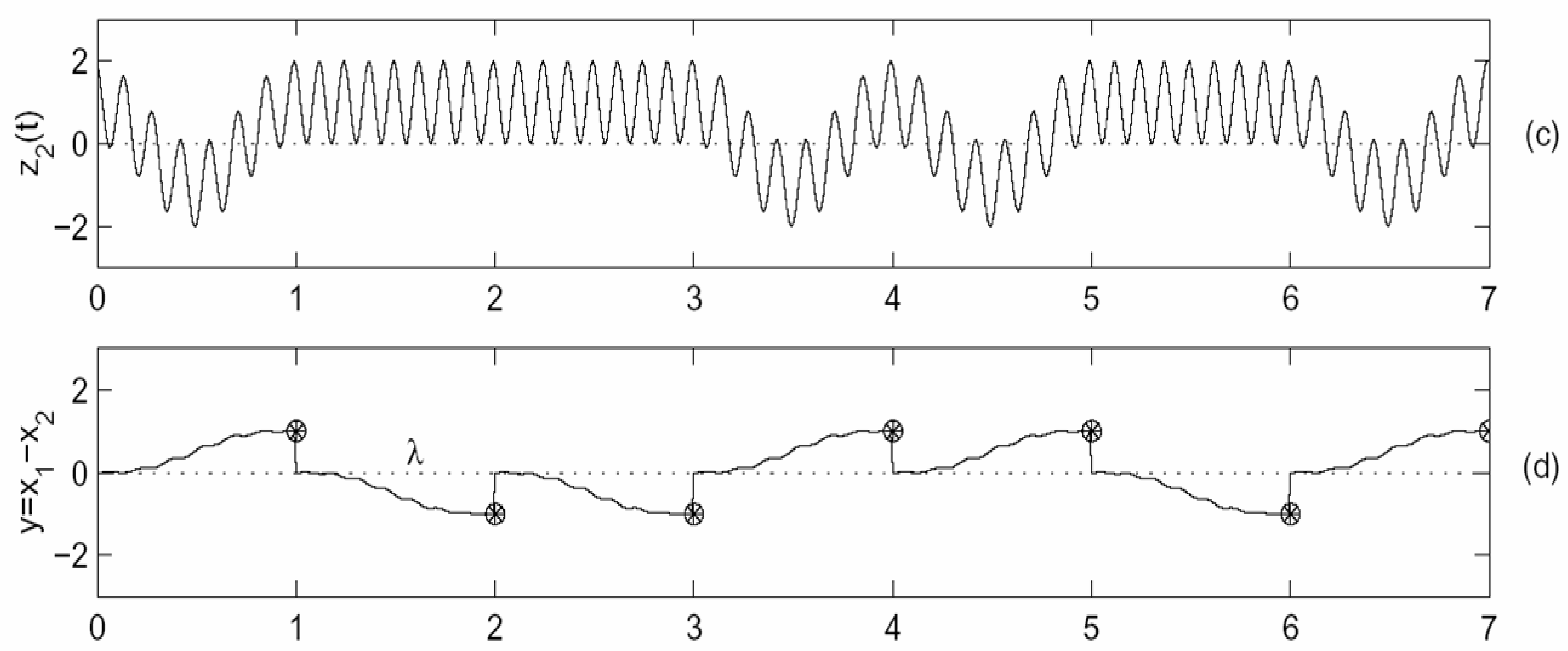


Fig. 5.4-2 Signals for an FSK system

(c) the multiplication output $z_2(t) = s(t)\sqrt{2/T_b}\cos(2\pi f_2 t)$

(d) the difference between two outputs of the integrators and the corresponding sampling points.

- If we want to detect $\phi_1(t)$, we multiply the received signal by $\phi_1(t)$ and integrate.
  → $\phi_2(t)$ part will disappear totally and only the coefficient of $\phi_1(t)$ remains.

- Since only one of the coefficients is nonzero, we can easily see whether 1 or 0 is transmitted.

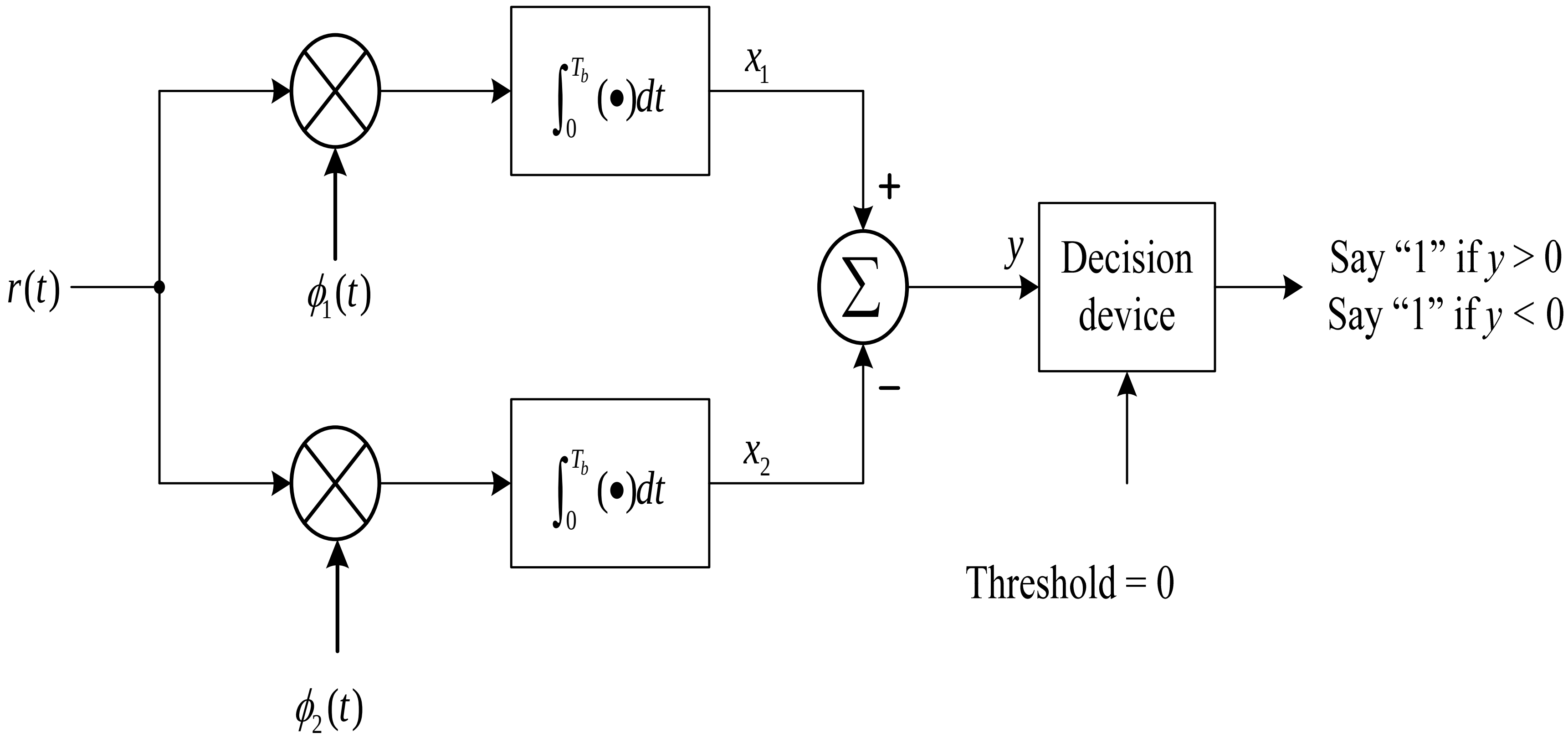


Fig. 5.4-3 A coherent detector for an FSK system.

- Fig. 5.4-3:
- The correlator outputs are **subtracted**, one from the other, and the resulting difference, y, is compared with a threshold of zero.

- If $y > 0$, the receiver decides in favor of 1.
  If $y < 0$, the receiver decides in favor of 0.

- Noiseless case:

  The received signal is exactly the transmitted signal.

- If the signal (representing 1) was transmitted, the outputs x_1 and x_2 of the correlators can be expressed as:

$$x_1 = \int_0^T \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_1 t) dt = \sqrt{E_b}$$

$$x_2 = \int_0^T \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_2 t) dt = 0. \qquad (5.4\text{-}9)$$

- If the signal (representing 0) was transmitted, the outputs of the correlators can be expressed as:

$$x_1 = \int_0^T \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_1 t) dt = 0$$

$$x_2 = \int_0^T \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_2 t) dt = \sqrt{E_b}. \quad (5.4\text{-}10)$$

- If $s_1(t)$ was transmitted, $y = x_1 - x_2 > 0$

  If $s_2(t)$ was transmitted, $y = x_1 - x_2 < 0$

- Fig. 5.4-2(b):
- The received signal multiplied by carrier with $f_1 = 3$ Hz and the result is
$z_1(t) = s(t) \times \sqrt{2/T_b} \cos(2\pi f_1 t)$.

- Fig. 5.4-2(c):
- The received signal multiplied by carrier with $f_2 = 4$ Hz and the result is
$z_2(t) = s(t) \times \sqrt{2/T_b} \cos(2\pi f_2 t)$

- The decision device based on the sampled values makes a decision of "**1001101**" which is exactly the same as the transmitted sequence.

- This FSK system is based upon the basic concept that we are transmitting two signals which are **orthonormal** to each other and form a basis.

- If we transmit two signals (1,0) and (0,1):
- These two vectors form a basis.

  → the inner product of (1,0) and (0,1) is 0

  and $\left|(1,0)\right| = \left|(0,1)\right|$.

- **Detect signals**:
- use the inner product.

- Ex: When we perform an inner product of the received signal with (1,0):

→If the transmitted signal is (1,0), the resulting inner product is 1.
If the transmitted signal is (0,1), the resulting inner product is 0.

- In Equation (5.4-3), $\phi_1(t)$ and $\phi_2(t)$ are two **orthonormal** basis functions.

- The signal points $s_1(t)$ and $s_2(t)$ can be expressed as:

$$s_i(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$$

for i = 1, 2 and $0 \leq t < T$. (5.4-11)

- Fig. 5.4-4:
- A coherent binary FSK system is characterized by having a signal space which is two-dimensional with two message points.

- The two message points are defined by $s_1 = (\sqrt{E_b}, 0)$ and $s_2 = (0, \sqrt{E_b})$.

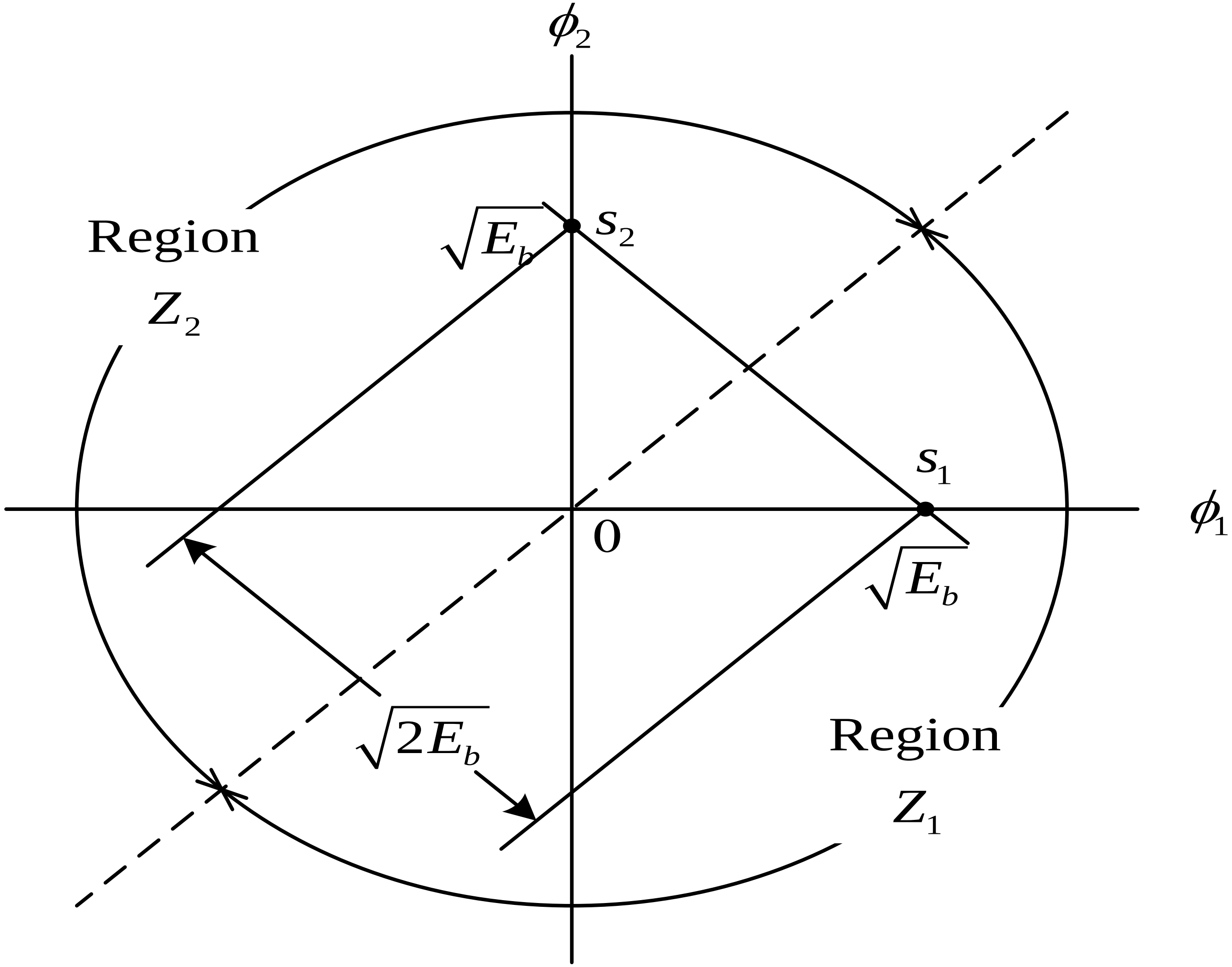


Fig. 5.4-4 Signal space diagram for binary FSK.

- Fig. 5.4-4:
- The decision boundary is characterized by the line with $x_1 - x_2 = 0$.
- If $x_1 - x_2 > 0$ (the received signal point lies in region Z_1) , the decision device makes a decision of 1.
- If $x_1 - x_2 < 0$ (the received signal point lies in region Z_2) , the decision device makes a decision of 0.

- The bandwidth of the FSK system is $2/T_b$.

- By Figures 5.2-4 ,5.3-3, and 5.4-4:
- The distances between two signal points for the binary ASK, BPSK, and FSK systems are $\sqrt{2E_b}$,$2\sqrt{E_b}$ and $\sqrt{2E_b}$,respectively.

- Under the same transmitted bit energy and the same level of noise corruption, the **BPSK** system gives the lowest bit error rate since BPSK system has the largest distance between two signal points.

5.5 Quadriphase-Shift Keying (QPSK)

- The important goals in the design of a digital communication system are:

  1. provide a reliable communication, i.e., achieve a very low probability of error.
  2. the efficient utilization of channel bandwidth.

- 113QPSK transmit two bits simultaneously during one signaling interval T .

- QPSK double the transmitted bit rate without increasing the transmitted bandwidth.

- The two transmitted bits denoted as m_1 and m_2. They are separated from a single bit stream m .

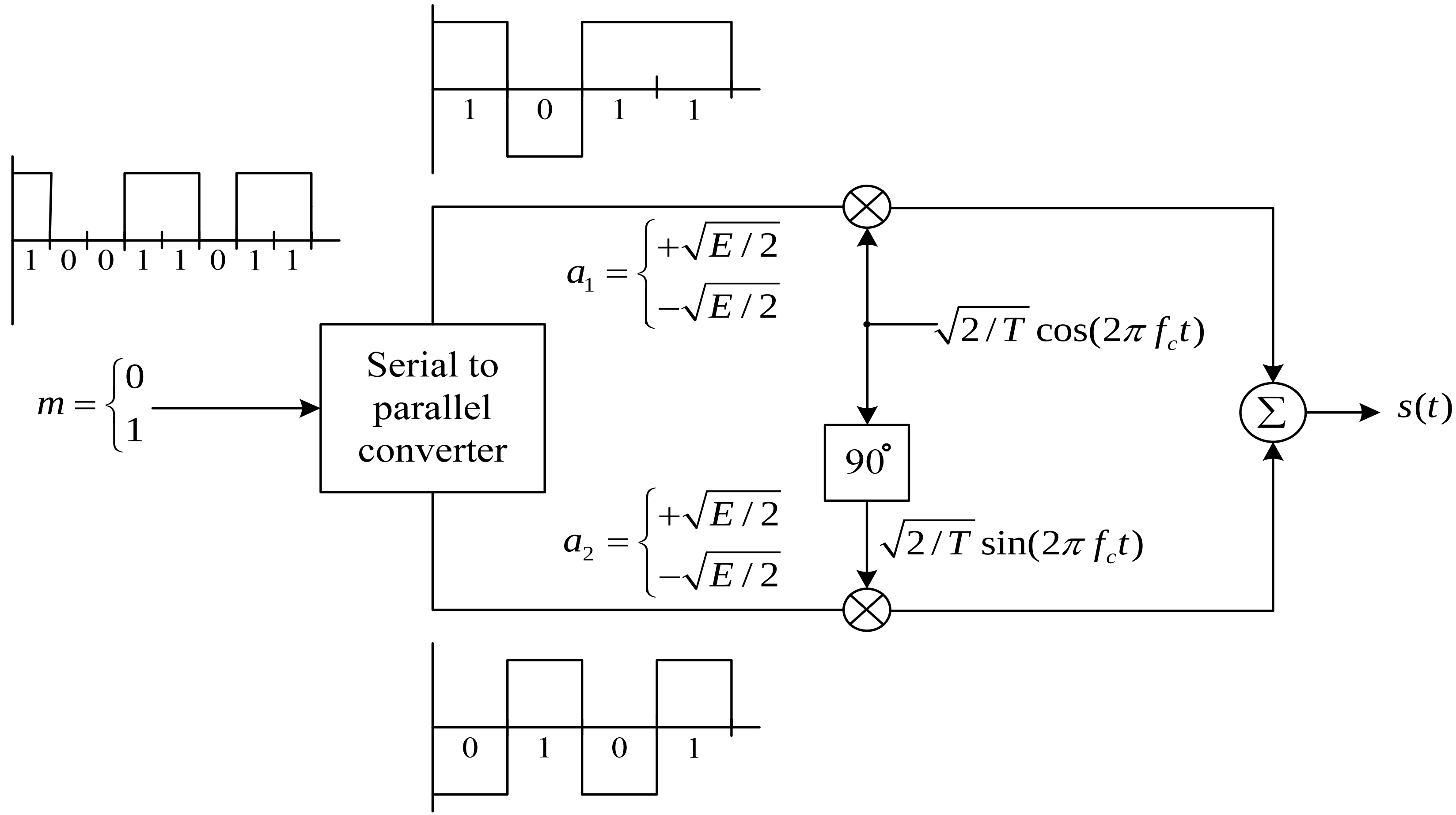


Fig. 5.5-1 A schematic diagram of QPSK.

- Fig. 5.5-1:
- m_1 represents even bits of m and m_2 represents odd bits of m .
- m_1 will go up and m_2 will go down.
- The following rules will follow:

1. m_1 will trigger signal a_1 and m_2 will trigger signal a_2.

2. If m_1 is equal to 1(0), a_1 is set to be

$+\sqrt{E/2}\left(-\sqrt{E/2}\right)$.

3. If m_2 is equal to 1(0), a_2 is set to be

$+\sqrt{E/2}\left(-\sqrt{E/2}\right)$.

4. a_1 (a_2) will be multiplied by

$\sqrt{2/T}\cos\left(2\pi f_c t\right)\left(\sqrt{2/T}\sin\left(2\pi f_c t\right)\right)$.

5. The signal transmitted at any time *t* is

$$s(t) = a_1 \sqrt{\frac{2}{T}} \cos(2\pi f_c t) + a_2 \sqrt{\frac{2}{T}} \sin(2\pi f_c t) \qquad (5.5\text{-}1)$$

- The relationship between (m_1, m_2) and (a_1, a_2) is as shown in Table 5.5-1.

- Table 5.5-1 The mapping of (m_1, m_2)'s to (a_1, a_2)'s

(m_1, m_2)	(a_1, a_2)
$(1, 1)$	$\left(\sqrt{E/2}, \sqrt{E/2}\right)$
$(1, 0)$	$\left(\sqrt{E/2}, -\sqrt{E/2}\right)$
$(0, 1)$	$\left(-\sqrt{E/2}, \sqrt{E/2}\right)$
$(0, 0)$	$\left(-\sqrt{E/2}, -\sqrt{E/2}\right)$

- In Equation (5.5-1), we can easily show that

$$s(t) = \sqrt{\frac{2}{T}} r \cos(2\pi f_c t - \theta) \qquad (5.5\text{-}2)$$

where $r = \sqrt{a_1^2 + a_2^2}$ and $\theta = \tan^{-1} \frac{a_2}{a_1}$

- $|a_1| = |a_2| = \sqrt{\frac{E}{2}},\ r = \sqrt{E}$.
- θ is related to the values of s_{i1} and s_{i2} .

- There are four possible values of θ, corresponding to four distinct combinations of a_1 and a_2.

- Every θ assumes an integer multiplication of $\frac{\pi}{4}$.

- From Equation 5.5-2:

  There are four different $s(t)$'s, denoted as $s_i(t)$ for i = 1, 2, 3, 4.

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + (2i-1)\frac{\pi}{4}\right) \quad \text{for} \quad 0 \le t < T. \tag{5.5-3}$$

- Ex: $m_1 = 1$ and $m_2 = 0$

  → $i = 1$, $a_1 = \sqrt{\frac{E}{2}}$ and $a_2 = -\sqrt{\frac{E}{2}}$

$$\therefore s_1(t) = \sqrt{\frac{E}{T}}(\cos(2\pi f_c t) - \sin(2\pi f_c t))$$

$$= \sqrt{\frac{E}{T}}(\sqrt{2}(\frac{\sqrt{2}}{2}\cos(2\pi f_c t) - \frac{\sqrt{2}}{2}\sin(2\pi f_c t))$$

$$= \sqrt{\frac{2E}{T}}(\cos(\frac{\pi}{4})\cos(2\pi f_c t) - \sin(\frac{\pi}{4})\sin(2\pi f_c t)) \qquad (5.5\text{-}4)$$

$$= \sqrt{\frac{2E}{T}}\cos(2\pi f_c t + \frac{\pi}{4}).$$

- Expand Equation (5.5-3) as follows:

$$s_1(t) = \sqrt{\frac{2E}{T}}\cos\left((2i-1)\frac{\pi}{4}\right)\cos(2\pi f_c t) - \sqrt{\frac{2E}{T}}\sin\left((2i-1)\frac{\pi}{4}\right)\sin(2\pi f_c t)$$

(5.5-5)

- By Equation (5.5-5):

$$a_1 = +\sqrt{E}\cos\left(\left(2i-1\right)\frac{\pi}{4}\right) \quad \text{and} \quad a_2 = -\sqrt{E}\sin\left(\left(2i-1\right)\frac{\pi}{4}\right) \tag{5.5-6}$$

- $s_i(t)$ for $i = 1, 2, 3, 4,$ corresponds to $m_1 m_2 = 10, 00, 01, 11,$ respectively.
  (see Table 5.5-1.)

- Fig. 5.5-1:
- The stream of input data is **10011011**.
- The upper stream (odd numbered bits) is 1011 and the lower stream (even numbered bits) is 0101.
- At each time slot, one bit from the upper stream is combined with a corresponding bit from the lower stream.

  → output data stream is (10, 01, 10, 11).

- From Table 5.5-2:
- The signals sent for these four time slots are $s_1(t), s_3(t), s_1(t), s_4(t)$.
- signal $s_i(t)$ is determined by m_1 and m_2.
- Rewrite Equation (5.5-1) as follows:

$$s(t) = A\cos(2\pi f_c t) + B\sin(2\pi f_c t)$$

- The job of demodulation is to detect A and B..

Table 5.5-2 Signal-space Characterization of QPSK.

	Input Digit $(m_1 m_2)$	Phase	a_1	a_2	$s_i(t)$
$s_1(t)$	10	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$	$\sqrt{2E/T}\cos(2\pi f_c t+\pi/4)$
$s_2(t)$	00	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$	$\sqrt{2E/T}\cos(2\pi f_c t+3\pi/4)$
$s_3(t)$	01	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$	$\sqrt{2E/T}\cos(2\pi f_c t+5\pi/4)$
$s_4(t)$	11	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$	$\sqrt{2E/T}\cos(2\pi f_c t+7\pi/4)$

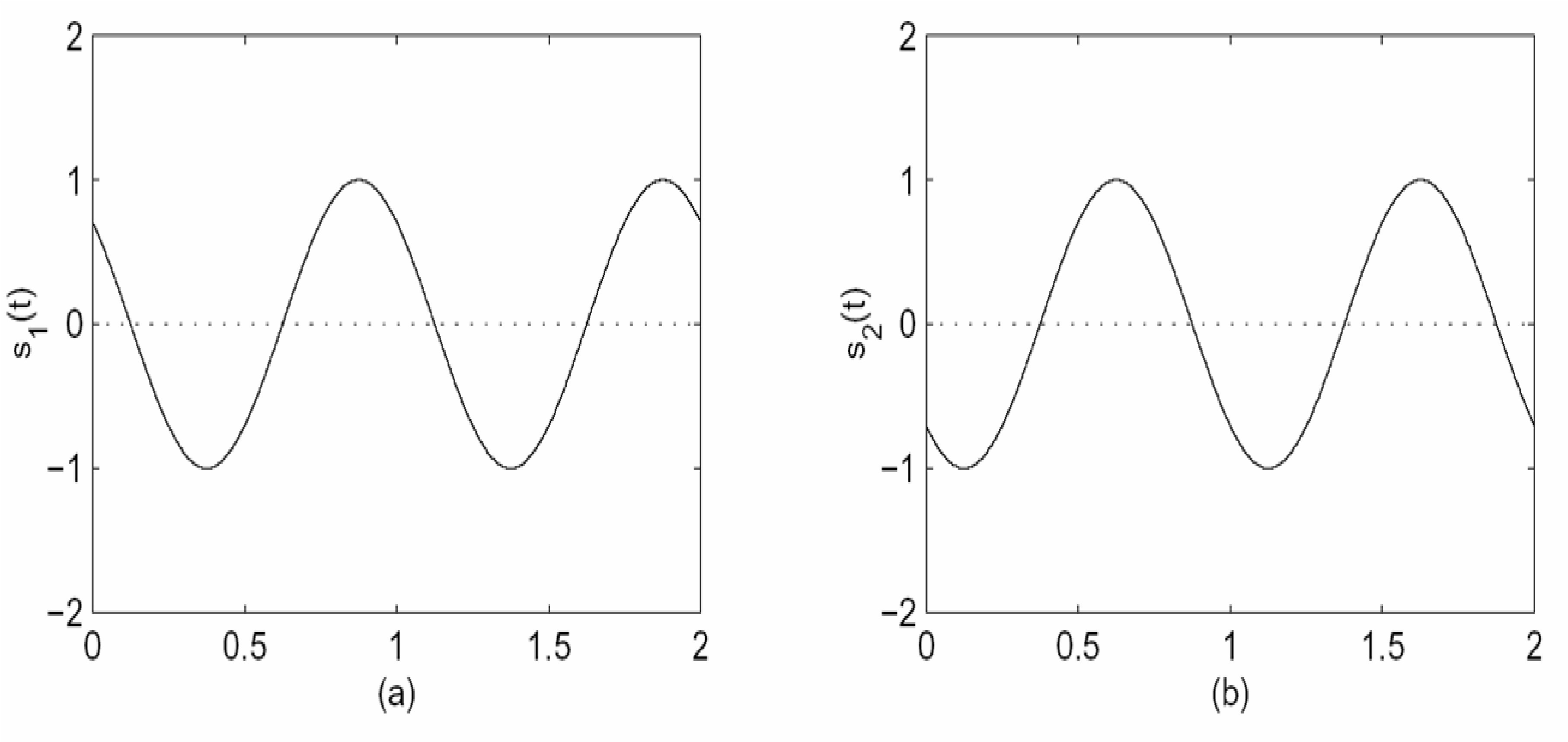


Fig. 5.5-2

The four QPSK signals .for $i = 1,2,3,4,$ where f_c = 1

(a) signal of $s_1(t)$, (b) signal of $s_2(t)$

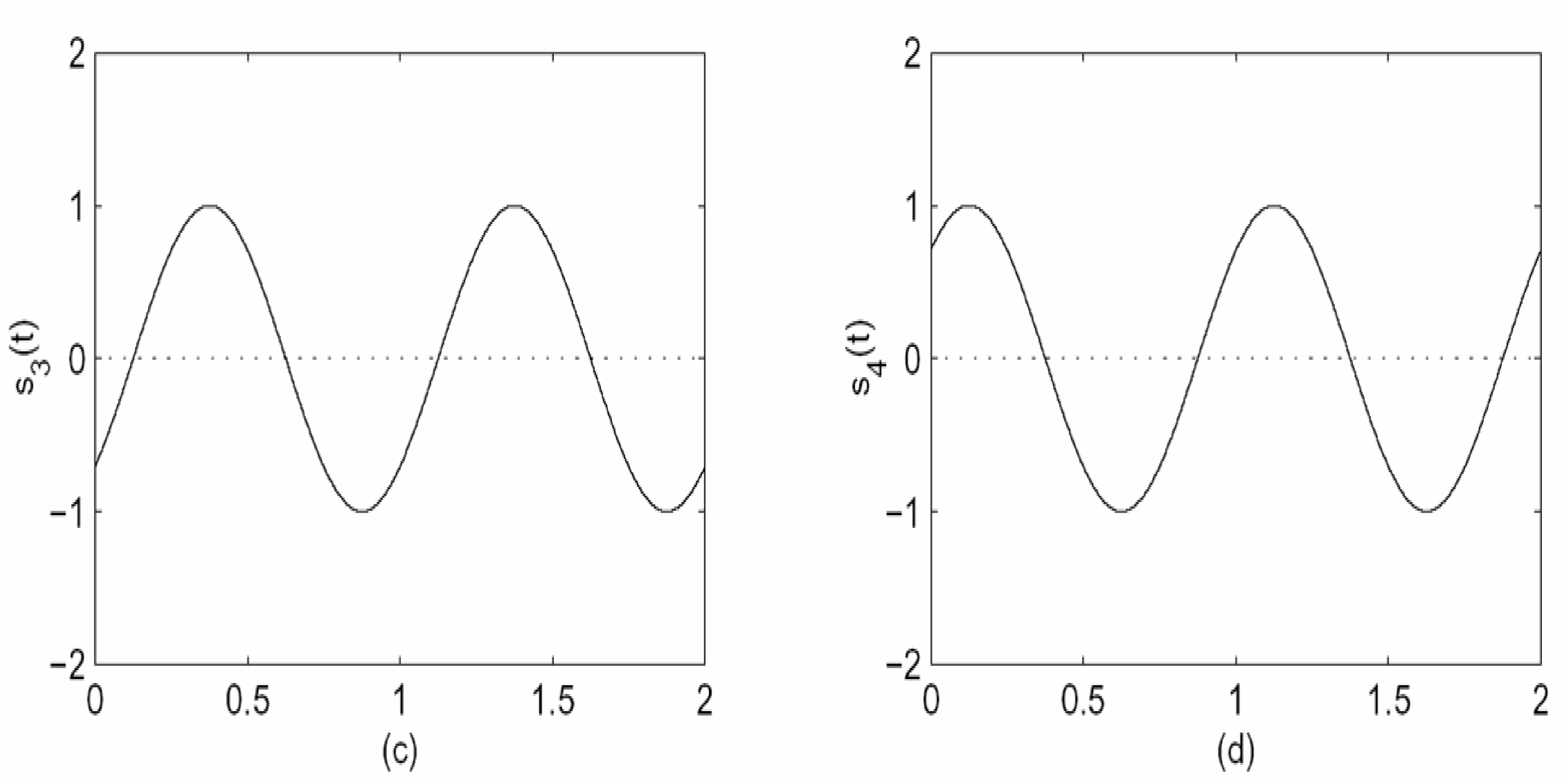


Fig. 5.5-2

The four QPSK signals .for $i = 1,2,3,4,$ where $f_c = 1$

(a) signal of $s_3(t)$, (b) signal of $s_4(t)$

- Basic principle of QPSK demodulation
- $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ are orthogonal.
- To detect A, we multiply $s_i(t)$ by $\cos(2\pi f_c t)$ and integrate.

  →**eliminate B and only A remains**.
- To detect B, we multiply $s_i(t)$ by $\sin(2\pi f_c t)$ and integrate.

  →**eliminate A and only B remains**.

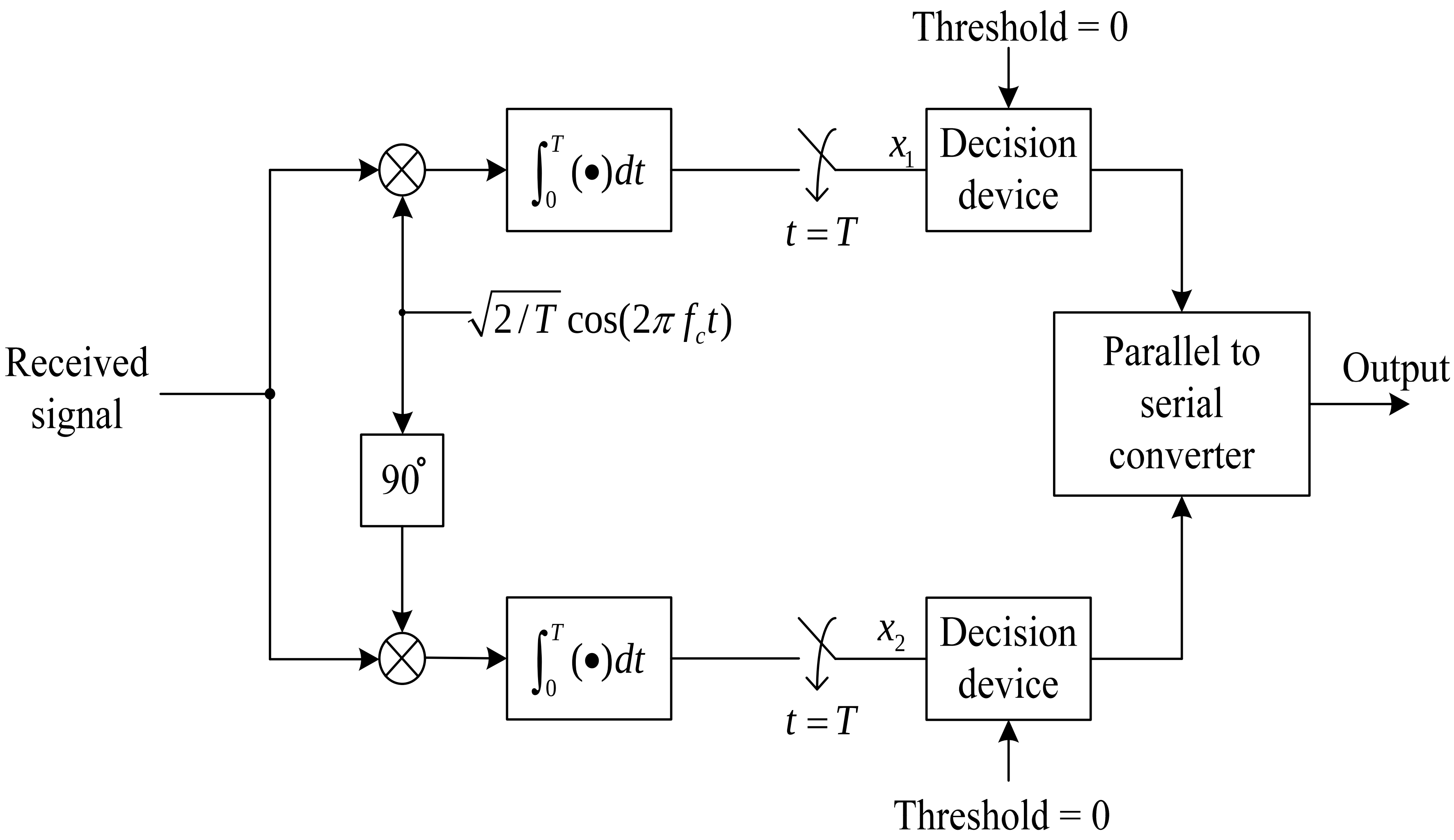


Fig. 5.5-3 Demodulator for QPSK.

- Fig. 5.5-3:
- The received signal is first multiplied by two sinusoids $\sqrt{2/T}\cos(2\pi f_c t)$ and $\sqrt{2/T}\sin(2\pi f_c t)$
- The results are then integrated respectively by two integrators.
- The outputs of the integrators are sampled at $t = T$.

- For noiseless case, the resulting sampled output x_1 can be expressed as:

$$x_1 = \int_0^T \left(a_1 \sqrt{\frac{2}{T}} \cos(2\pi f_c t) + a_2 \sqrt{\frac{2}{T}} \sin(2\pi f_c t) \right) \times \sqrt{\frac{2}{T}} \cos(2\pi f_c t) dt$$

$$= a_1 \frac{2}{T} \left(\int_0^T \cos^2(2\pi f_c t) dt \right) + a_2 \frac{2}{T} \left(\int_0^T \sin(2\pi f_c t) \cos(2\pi f_c t) dt \right)$$

$$= a_1 \frac{2}{T} \left(\int_0^T \frac{1}{2} + \frac{1}{2} \cos(4\pi f_c t) dt \right) + a_2 \frac{2}{T} \left(\int_0^T \frac{1}{2} \sin(4\pi f_c t) dt \right).$$

(5.5-7)

- Replacing $f_c = n_c / T$ in the above equation where n_c is a positive integer:

$$x_1 = a_1 \frac{2}{T}\left(\frac{t}{2} + \frac{1}{8\pi(n_c)/T}\sin\left(4\pi\frac{n_c}{T}t\right)\right)\Bigg|_0^T + a_2\frac{2}{T}\left(-\frac{1}{8\pi(n_c)/T}\cos\left(4\pi\frac{n_c}{T}t\right)\right)\Bigg|_0^T$$

$$= a_1\frac{2}{T}\left(\left(\frac{T}{2}+0\right)-(0+0)\right)+a_2\frac{2}{T}\left(-\frac{1}{8\pi(n_c/T)}+\frac{1}{8\pi(n_c/T)}\right).$$

(5.5-8)

- $\sin(4\pi n_c) = 0$ and $\cos(4\pi n_c) = 1$ for any positive integer n_c.

→ $x_1 = a_1$ (5.5-9)

- Similarly,

$$x_2 = \int_0^T \left(a_1 \sqrt{\frac{2}{T}} \cos(2\pi f_c t) + a_2 \sqrt{\frac{2}{T}} \sin(2\pi f_c t) \right) \times \sqrt{2/T} \sin(2\pi f_c t) dt = a_2.$$

(5.5-10)

- The original messages a_1 and a_2 can be separated at the receiver and can be detected independently.

- Decision devices:
- for upper channel:

If $x_1 > 0$, decide $m_1 = 1$.

If $x_1 < 0$, decide $m_1 = 0$.

- for lower channel:

  If $x_2 > 0$, decide $m_2 = 1$.

  If $x_2 < 0$, decide $m_2 = 0$.

- Finally, these two binary sequences are combined in a parallel-to-serial converter to reproduce the original binary sequence at the transmitter input.

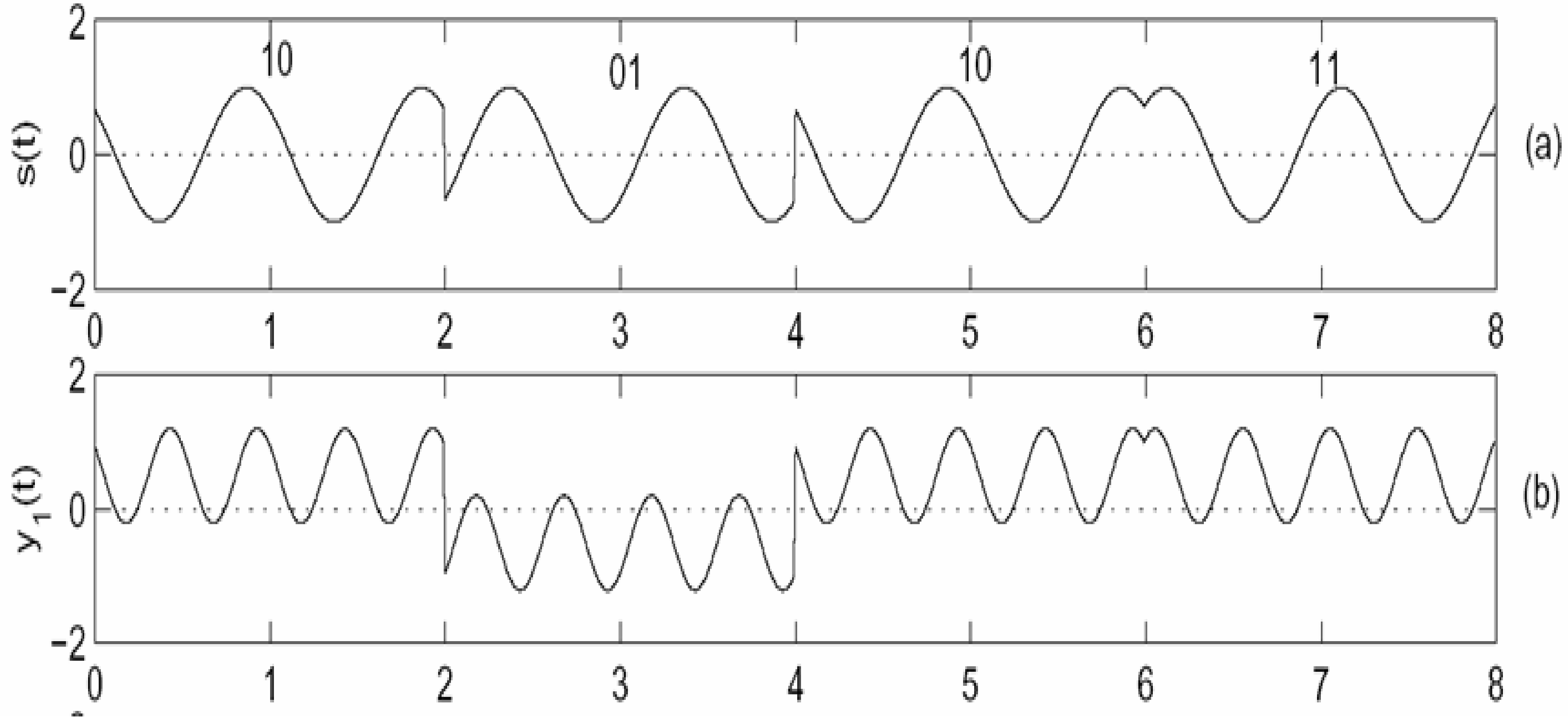


Fig. 5.5-4 Signals of QPSK for $f_c = 1$

(a) the transmitted signal,

(b) the signal $y_1(t) = s(t) \times \sqrt{2/T} \cos(2\pi f_c t)$

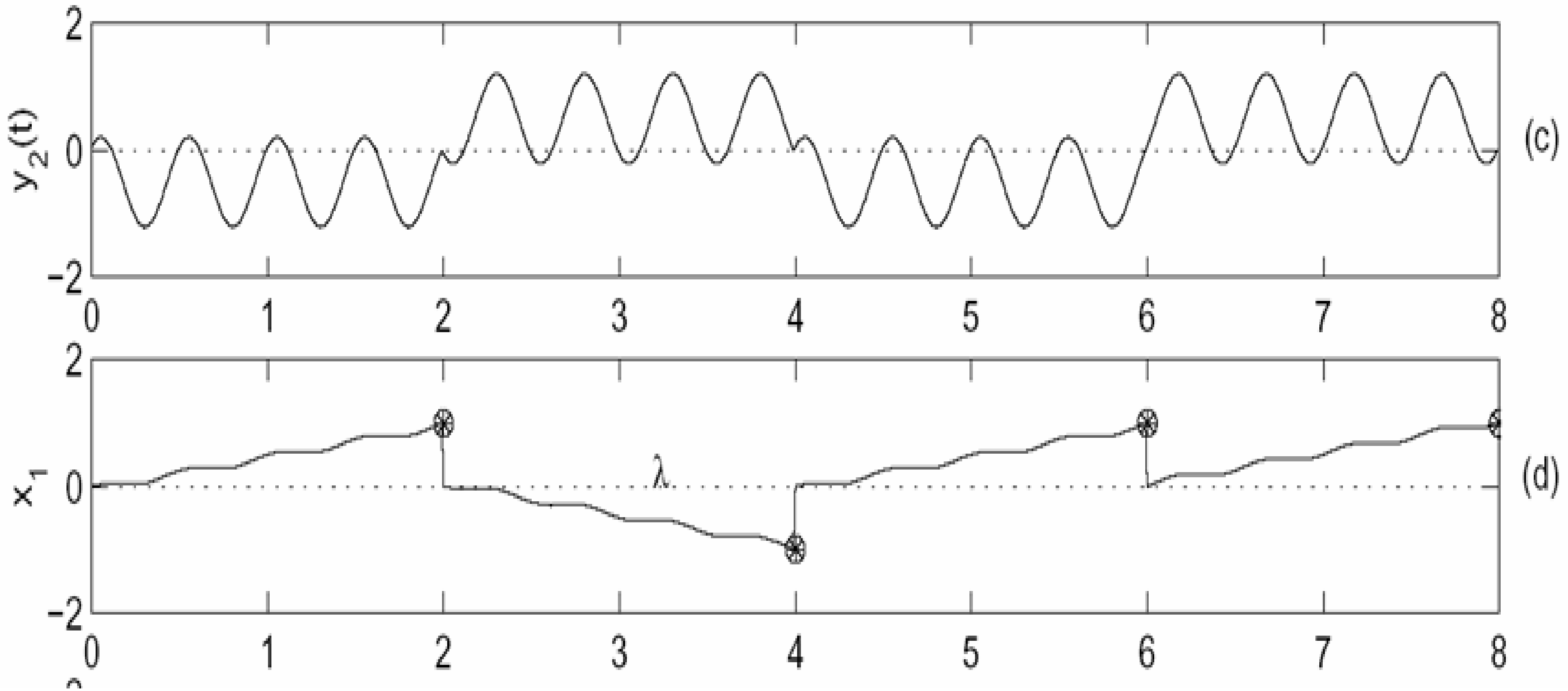


Fig. 5.5-4 Signals of QPSK for $f_c = 1$

(c) the signal of $y_2(t) = s(t) \times \sqrt{2/T}\,\sin(2\pi f_c t)$

(d) the output of the upper integrator and the corresponding sampling points

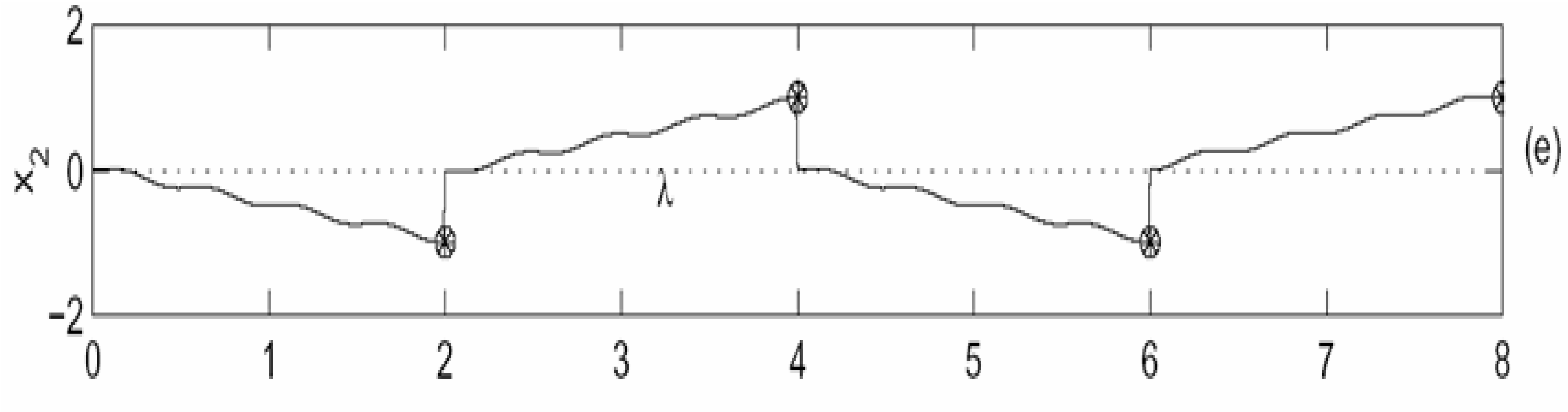


Fig. 5.5-4 Signals of QPSK for $f_c = 1$

(e) the output of the lower integrator and the corresponding sampling points.

- Fig. 5.5-4:
- Transmitted signal is given in Fig. 5.5-4 (a) which is a modulated signal with the input bit stream “**10011011**”, where , f_c = 1 Hz and T = 2 sec.

- At the receiver, the signals after the multiplication of carriers denoted by $y_1(t) = s(t) \times \sqrt{2/T} \cos(2\pi f_c t)$ and $y_2(t) = s(t) \times \sqrt{2/T} \sin(2\pi f_c t)$ are given in Fig. 5.5-4(b) and Fig. 5.5-4(c) respectively.

- The outputs from the integrators and the corresponding sampling points x_1 and x_2 are given in Fig. 5.5-4(d) and Fig. 5.5-4(e), respectively.

- Based on the sampled values, the decision output after the **parallel-to-serial** converter is “**1001101**” which is exactly the same as the transmitted sequence.

- Consider the input datum is **1 0**.

  According to Table 5.5-1:

$$s(t) = \sqrt{\frac{2E}{T}} \cos(2\pi t + \frac{\pi}{4}).$$

- Thus,

$$y_1(t) = \sqrt{\frac{2E}{T}}\cos(2\pi t + \frac{\pi}{4})\sqrt{\frac{2}{T}}\cos(2\pi t)$$

$$= \frac{2\sqrt{E}}{T}(\frac{1}{2})(\cos(4\pi t + \frac{\pi}{4}) + \cos(\frac{\pi}{4}))$$

$$= \frac{\sqrt{E}}{T}(\cos(4\pi t + \frac{\pi}{4}) + \cos(\frac{\pi}{4}))$$

$$= \frac{\sqrt{E}}{T}(2\cos^2(2\pi t + \frac{\pi}{8}) - 1 + \frac{\sqrt{2}}{2})$$

$$= \frac{\sqrt{E}}{T}(2\cos^2(2\pi t + \frac{\pi}{8}) - 1 + 0.707)$$

$$= \frac{\sqrt{E}}{T}(2\cos^2(2\pi t + \frac{\pi}{8}) - 0.293).$$

- For this case, $y_1(t)$ is positive for most time, as shown in Fig. 5.5-4(b).

- **Basic principle of the QPSK system**:
- Mixes two bits together and transmits them at the same time.

- They can be detected correctly because there are two orthonormal functions $\phi_1(t)$ and $\phi_2(t)$ contained in the expansion of $s_i(t)$.

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t) \qquad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t) \qquad 0 \le t \le T. \qquad (5.5\text{-}11)$$

- It is easy to verify that $\phi_1(t)$ and $\phi_2(t)$ are orthonormal basis functions:

$$\int_0^T \phi_1(t)\,\phi_2(t)\,dt = 0 \qquad (5.5\text{-}12)$$

and

$$\int_0^T \phi_i^2(t)dt = 1 \quad \text{for } i=1,2. \quad (5.5\text{-}13)$$

- The transmitted signal can be expressed as:

$$s_i(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t) \qquad (5.5\text{-}14)$$

- Equation (5.5-14) :
- Demodulation of the QPSK system can be done by multiplying $s_i(t)$ by $\phi_1(t)$ and $\phi_2(t)$ and integrating over $[0, T]$.

- In the QPSK system, there are four vectors $s_i = (s_{i1}, s_{i2})$ for $i = 1,2,3,4$ which can be represented as signal points in the signal space diagram as shown in Fig. 5.5-5.

- The bandwidth of the system is $2/T = 2/2T_b$.
  →in terms of bit duration is $1/T_b$.

- The higher the bit rate, the larger the bandwidth.

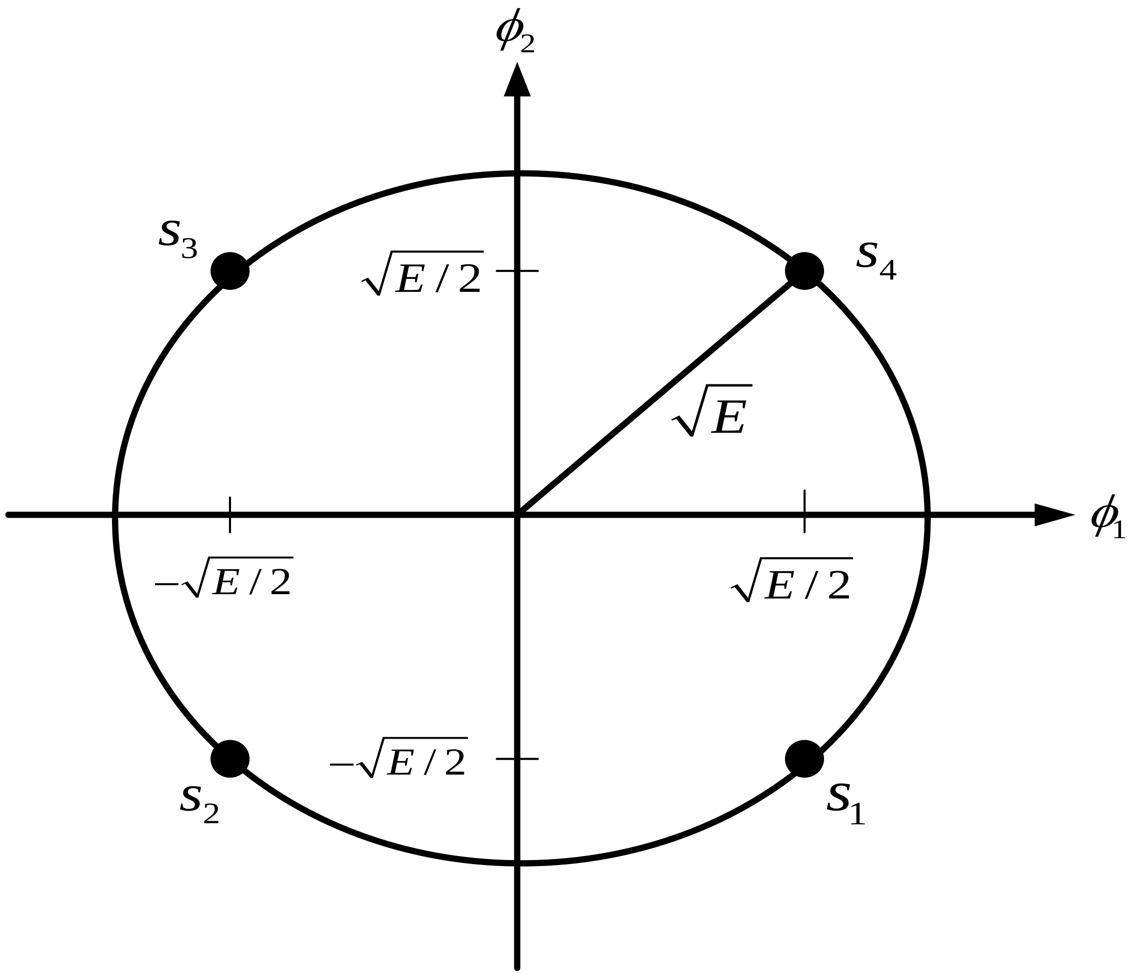


Fig. 5.5-5 Signal-space diagram of a QPSK system.

- It is possible to increase the transmitted bit rate by increasing the number of signals in the signal constellation.
- Ex: Double the number of QPSK signals, resulting in 8 signal points as shown in Fig. 5.5-6.

- For this QPSK system, there is a carrier frequency as the transmitted signal is a cosine function with frequency . The bandwidth of the system is still . Since for QPSK , the bandwidth of the system in terms of bit duration is . The higher the bit rate, the larger the bandwidth.

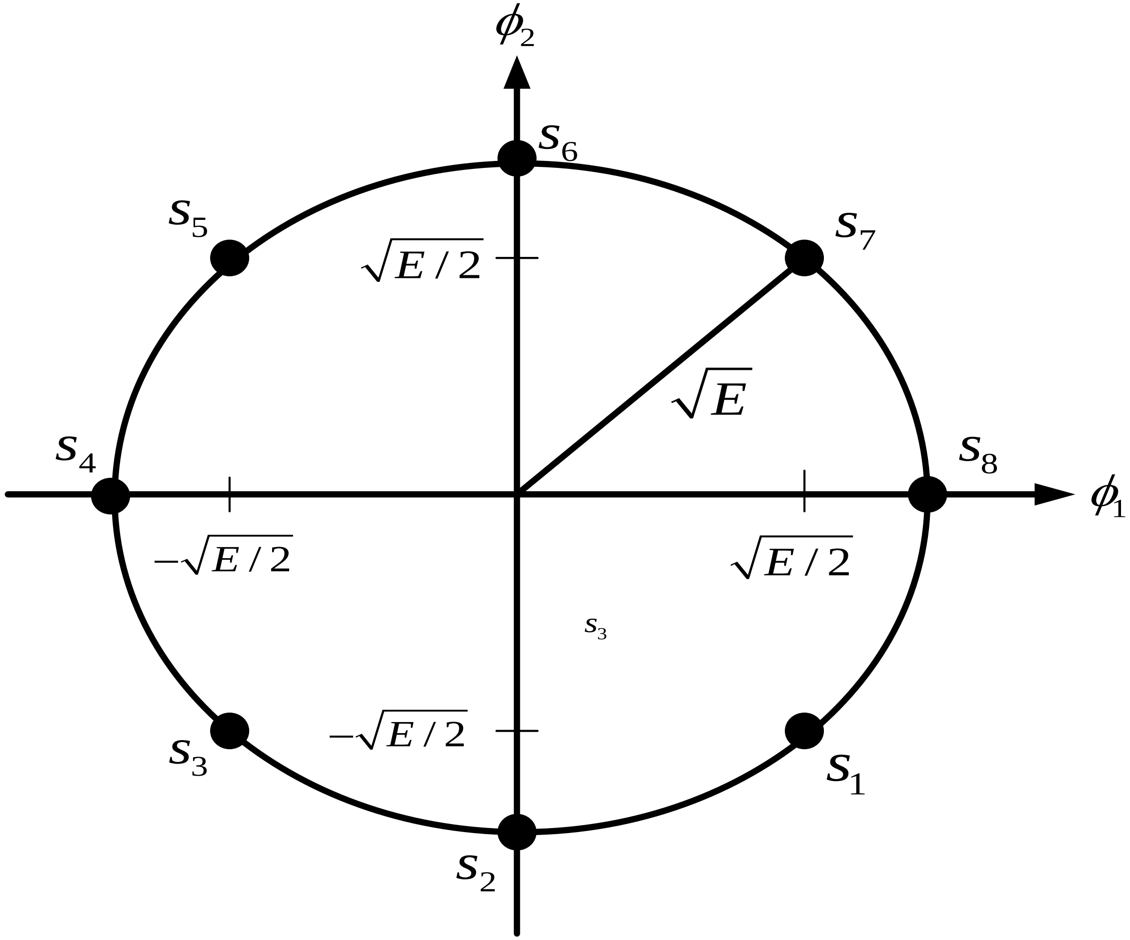


Fig. 5.5-6 Signal-space diagram of coherent 8PSK system.

- Fig. 5.5-6:
- The signal constellation is called **8PSK** which can transmit three bits at each signaling interval without increasing the transmitted bandwidth.

- Under the same transmitted signal energy per symbol:

  the distance between two nearest points is: 8PSK <QPSK

  →an 8PSK detector will make more errors than those of the QPSK systems.

- Increase the transmitted signal energy can achieve the same bit error rate as that of a QPSK system.

5.6 Quadrature Amplitude Modulation

- Fig. 5.6-1:
- BPSK, QPSK and 8PSK systems can be viewed as special cases of a more general class of digital modulation system, called the PSK.
- The signals sent are only different in their phases.

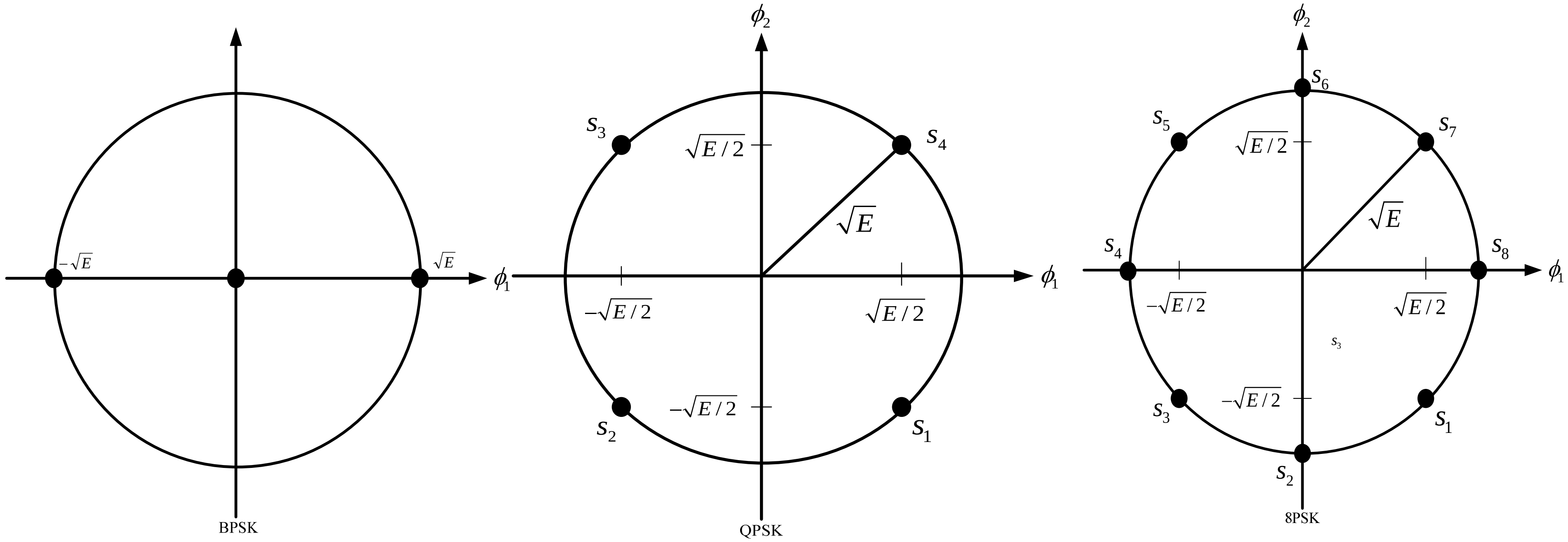


Fig. 5.6-1

The Signal-space diagrams for BPSK, QPSK and 8PSK

- **BPSK**: the phase difference is π .
  QPSK: the phase difference is $\pi/2$.
  8PSK: the phase difference is $\pi/4$.
- M-ary PSK system:
- Let $M = 2^k$, and k is the number of transmitted bits.

- The transmitted signal can be represented:

$$s_i(t)=\sqrt{\frac{2E}{T}}\cos\left(2\pi f_c t+(2i-1)\frac{\pi}{M}\right)$$

for $i=1,2,\ldots,M$ (5.6-1)

- Each $s_i(t)$ represents a possible state of sending *k* bits together.

- Similar to Equation (5.5-14), the signal can be equivalently written by

$$s_i(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$$

for $i = 1, 2, \ldots, M$ (5.6-2)

where $\phi_1(t)$ and $\phi_2(t)$ are defined in (5.5-10) as follows:

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$$

(5.6-3)

- It can be easily seen that

$$s_{i1} = \sqrt{E} \cos\left((2i-1)\frac{\pi}{M} \right)$$

$$s_{i2} = -\sqrt{E} \sin\left((2i-1)\frac{\pi}{M} \right)$$

(5.6-4)

- Plot the locations of the vectors (s_{i1}, s_{i2}) on a signal-space diagram.

- For PSK systems, the locations of the (s_{i1}, s_{i2}) vector are on a circle of radius $\sqrt{E}$. (see Fig. 5.5-5 and Fig. 5.5-6)

- In an M-ary PSK system, the message points are always on a circle of radius $\sqrt{E}$.

- 8PSK system:
- Three bits are sent together.

  →there are $2^3 = 8$ possible states.

- Label these 8 states arbitrarily by $i = 1, 2, \cdots, 8$.

- Each state is represented by (x_1, x_2, x_3) where each $x_i = 1$ or 0.

- Each (x_1, x_2, x_3) corresponds to a number if we use the binary numbering system.

i

- For instance (0,0,0) corresponds to 0, and (1,0,0) corresponds to 4 and (0,1,1) corresponds to 3.

- Suppose a vector corresponds to *j*.
- Label $i = j + 1$

Table 5.6-1 A possible labeling of (m_1, m_2, m_3)

(m_1, m_2, m_3)	i
(0,0,0)	1
(0,0,1)	2
(0,1,0)	3
(0,1,1)	4
(1,0,0)	5
(1,0,1)	6
(1,1,0)	7
(1,1,1)	8

- The modulating algorithm for the 8PSK system is as follows:

- **Step 1**: For the particular transmitted bits (m_1, m_2, m_3), find its corresponding index *i* from the table relating (m_1, m_2, m_3)'s to *i*'s.

- **Step 2**: Send the signal out according to Equation (5.6-1) by using the determined *i*.

- The received signal is in the form of Equations (5.6-2) and (5.6-3).

- $\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t)$ and $\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t)$ are orthogonal, s_{i1} and s_{i2} can be obtained by the concept of inner product.

- The index *i* can be found by using Equation (5.6-4).

- After *i* is determined, (m_1, m_2, m_3) can be determined through Table (5.6-1).

- The demodulating algorithm for the 8PSK system is as follows:

- **Step 1.** For the received signal $s_i(t)$, perform an inner product $\langle s_i(t), \cos(2\pi f_c t)\rangle$ $\langle s_i(t), \sin(2\pi f_c t)\rangle$ to determine S_{i1} (S_{i2}) based upon Equations (5.6-2) and (5.6-3).

- **Step 2.** Use the value of s_{i1} or s_{i2} to find the index *i* based upon Equation (5.6-4).
- From the table relating (m_1, m_2, m_3)'s to *i*'s, determine the corresponding (m_1, m_2, m_3).
- The above scheme can be used for any PSK system.

- When we send *k* bits together, we label each possible state $(x_1, x_2, \cdots, x_k)$ as a distinct *i* , $1 \le i \le 2^k$ and later when this state occurs, we use Equation (5.6-4) to determine s_{i1} and s_{i2}.

- If we send more bits together, we will have more points on the circle and a higher error rate.

- Now, introduce a method to send bits together which avoids above problem.
  → M-ary quadrature amplitude modulation
  (**M-QAM**) system.
- M-QAM system:
- The constraint expressed in Equation (5.6-1) is removed, and the components s_{i1} and s_{i2} are modulated independently.

- An M-ary QAM signal can be expressed:

$$s_i(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$$

for $i = 1, 2, \ldots, M$ (5.6-5)

where $\phi_1(t)$ and $\phi_2(t)$ are orthogonal.

- Two independent messages can be modulated over the amplitudes s_{i1} and s_{i2} of $\phi_1(t)$ and $\phi_2(t)$, respectively.

- The s_{i1} and s_{i2} can take values from a finite set of numbers.

- Ex: 16-QAM modulation

  s_{i1} and s_{i2} may take values from $\{-3,-1,+1,+3\}$.

- All the possible signal points (signal **constellation**) for the 16-QAM are shown in Fig. 5.6-2.

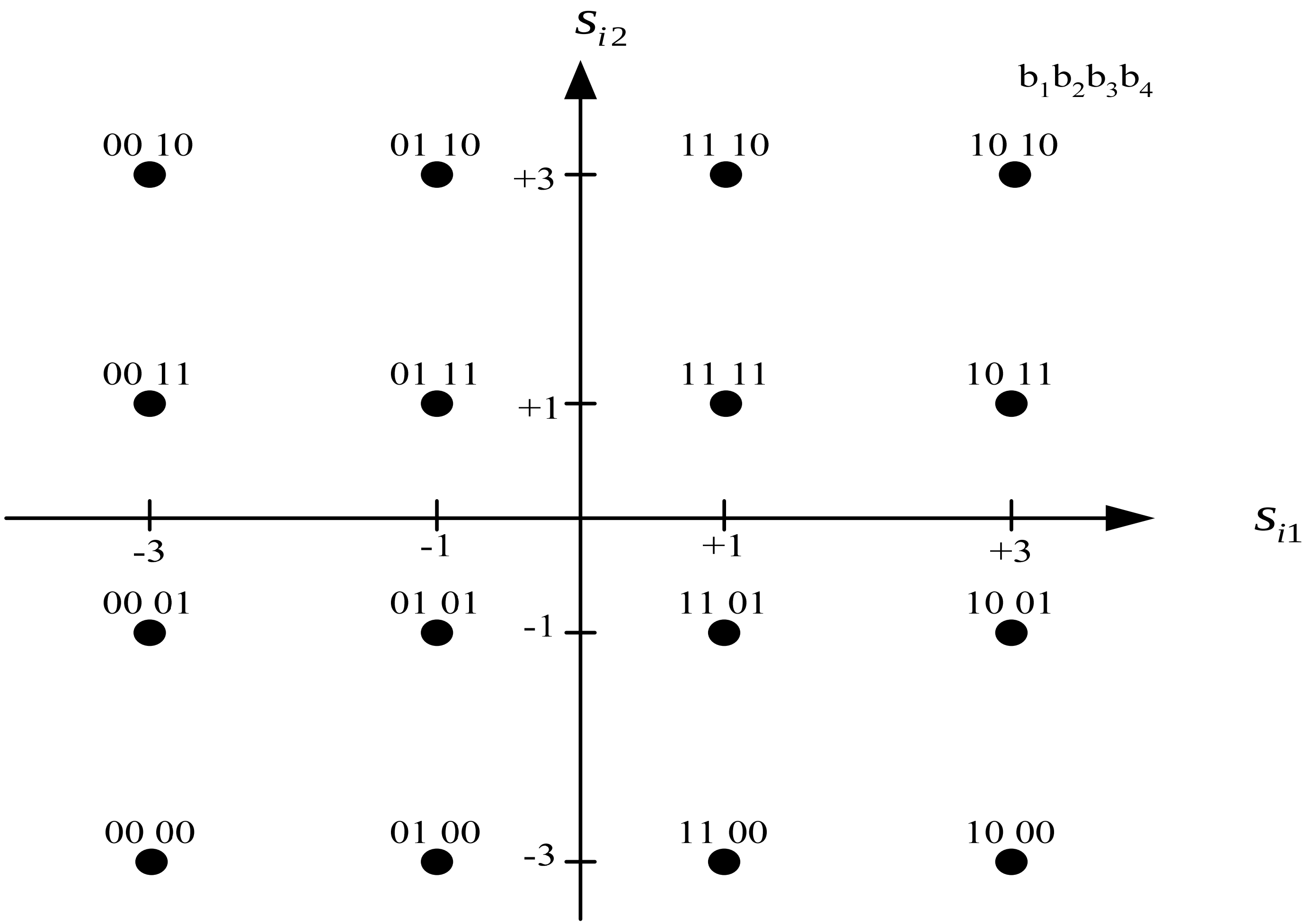


Fig. 5.6-2 A signal constellation for 16-QAM.

- Fig. 5.6-2:
- There are 16= 2^4 points in the signal constellation, the system is capable of transmitting 4 bits per signaling interval.

- The message bits are denoted by the vector (b_1, b_2, b_3, b_4) with $b_i \in \{0,1\}$.

- The message vector (b_1, b_2, b_3, b_4) is divided into two vectors (b_1, b_2) and (b_3, b_4) .

- The first vector (b_1, b_2) is mapped to s_{i1} , related to the first two bits, and the second vector is mapped to s_{i2} , related to the last two bits, according to Table 5.6-2.

Table 5.6-2 Mapping of bit patterns (b_1, b_2, b_3, b_4) to the signal points (s_{i1}, s_{i2})

(b_1, b_2)	s_{i1}	(b_3, b_4)	s_{i2}
(0,0)	-3	(0,0)	-3
(0,1)	-1	(0,1)	-1
(1,1)	1	(1,1)	1
(1,1)	3	(1,1)	3

- Extend the signal constellation to 64-QAM or even higher level of signal constellation, e.g., 256-QAM.
- Advantage for a large signal constellation: a larger number of bits can be sent together and the transmission data rate can be increased.

- If a **PSK** system is used, for a transmitter with fixed transmission power, the symbol error probability is increased by increasing the size of the signal constellation, since the distance between signal points is decreased due to a dense constellation.

- M-QAM system is more desirable because the constellation is not so dense as that in a PSK system.